On the impact of small-world on local search

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Motivation

The impact of *structure* – whatever it is – on search algorithms is dramatically relevant

- Identify most difficult instances (for a given technique)
- Understand *why* an instance is difficult
- Exploit this bit of information to choose the best solver, or a combination of solvers
- Evaluate the quality of benchmarks

Goal

Previous work [Walsh, 1999]

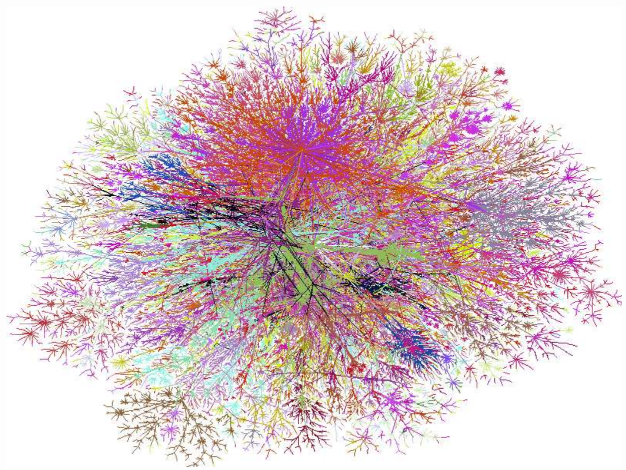
- CSP instances defined over 'small-world' graphs are harder to solve for complete algorithms

Question: What about local search behavior on small-world instances?

Outline

- Background: Complex networks
- *Structure* in CSPs
- Small-world SAT instances
- Experimental results
- Discussion

Complex networks



Complex networks

- System topology is crucial for understanding its dynamics
- Graph theory provides useful models
- Complex networks: emerging research field

Graphs as structure abstraction

- Entities represented as graph nodes
- relations $\leftrightarrow$ arcs
- Node: either one entity or an entire subsystem

Main characteristics

- Node degree (distribution, average, etc.)
- Diameter, characteristic path length et similia
- Clustering (i.e., cliquishness tendency)

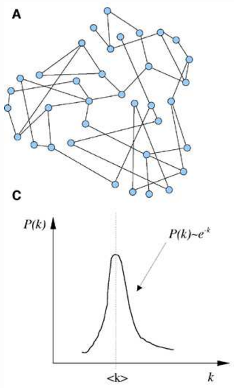
Random graphs

- First developed model for system structure
- Several important applications
- Random graphs fail to represent social and biological systems

Random graphs

- Node degree distribution: Poissonian (approx Normal)
- Characteristic path length: low
- Clustering: low

Random graphs



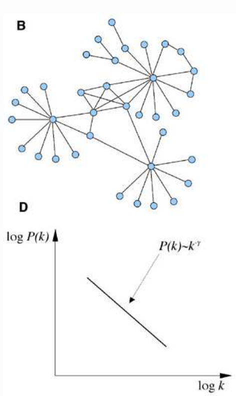
Scale-free networks

- Relations among individuals in a society (e.g., scientific collaborations)
- Web pages structure
- Internet structure
- . . .

Scale-free networks

- Node degree distribution:
  nodes with degree $k \sim k^{-\gamma}$ (γ parameter)
- Very few *hubs* (but not negligible) and many nodes with few connections
- Robust wrt random failures
- Sensitive to attacks

Scale-free networks



Scale-free networks formation

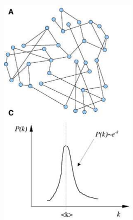
- *Growth*: older nodes has on average a higher number of connections
- *Preferential attachment*: new nodes are more likely to connect to nodes with higher degree (probability proportional to the degree)
- Model variants that take into account also the *fitness*

Small-world

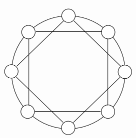
- Any pair of nodes connected by few hops (short characteristic path length)
- High degree of cliquishness (high clustering coefficient)
- Examples:
  - Social networks
  - World Wide Web
  - Scientific collaboration network
  - *C.Elegans* worm neural network

Characteristic length

Informally: average path length between any pair of nodes.

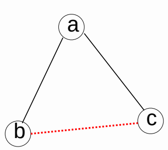


Random graphs $\longrightarrow$ short



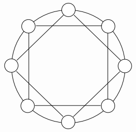
Grid graphs $\longrightarrow$ long

Clustering



Informally: it quantifies the probability that, given node a connected to b and c, there is an edge between b and c

Random graphs $\longrightarrow$ low



Grid graphs $\longrightarrow$ high

Structure

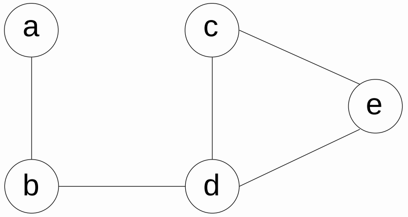
- Diverse meanings
- *Structure* vs. *random*
- Usually *real world* problems are said to be structured
- Attempts to define quantitative measures (entropy, compression ratio, etc.)

► Graph representation of relations among problem entities

SATgraphs

$$(a \vee \neg b) \wedge (b \vee d) \wedge (c \vee \neg d \vee \neg e)$$

$$\Downarrow$$



Remember the initial goal. . .

Previous work [Walsh, 1999]

- CSP instances defined over ‘small-world’ graphs are harder to solve for complete algorithms

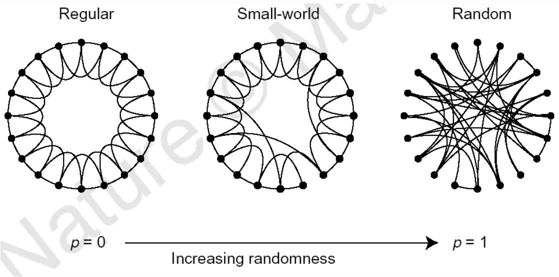
Question: What about local search behavior on small-world instances?

Experimental issues

- Small-world SAT instances
  - Procedure to generate instances
  - Measuring ‘small-world’ property
- Attacking the benchmark with local search algorithms
  - GSAT
  - WalkSAT
  - ILS-SAT

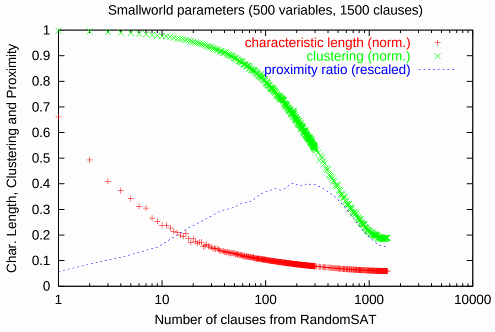
Small-world SAT

- Morphing between a lattice SAT instance and a random SAT instance. [Gent et al., 1999]

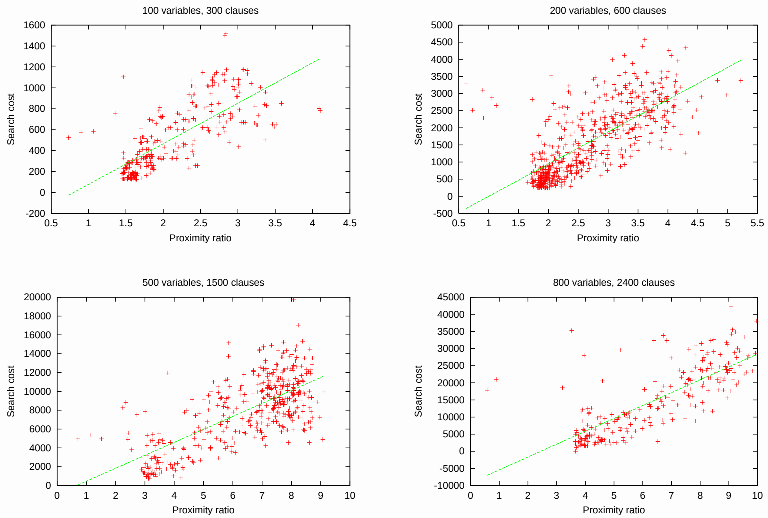


Small-world SAT

Length, clustering and proximity ratio (normalized ratio clustering/length)



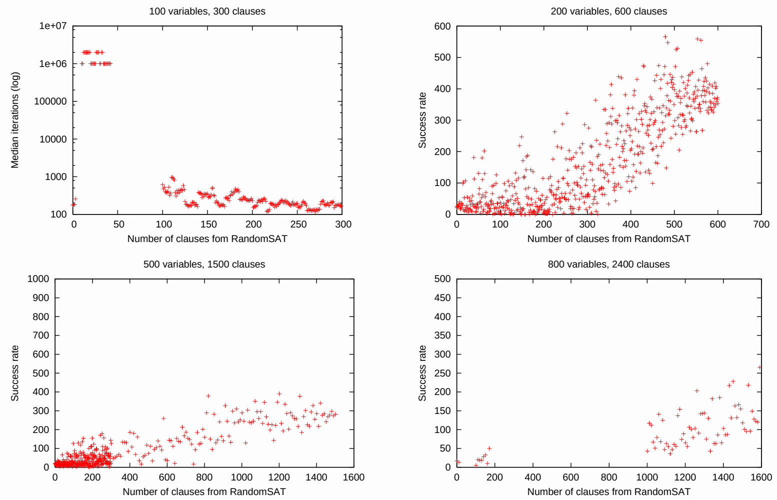
Complete algorithm



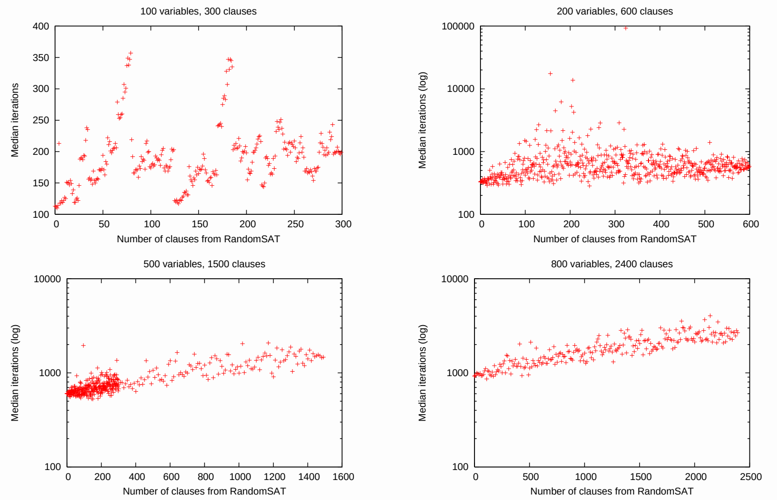
Outline of the results

- No common behavior across different algorithms
- ‘Mild’ tendency of small-world and hardness correlation

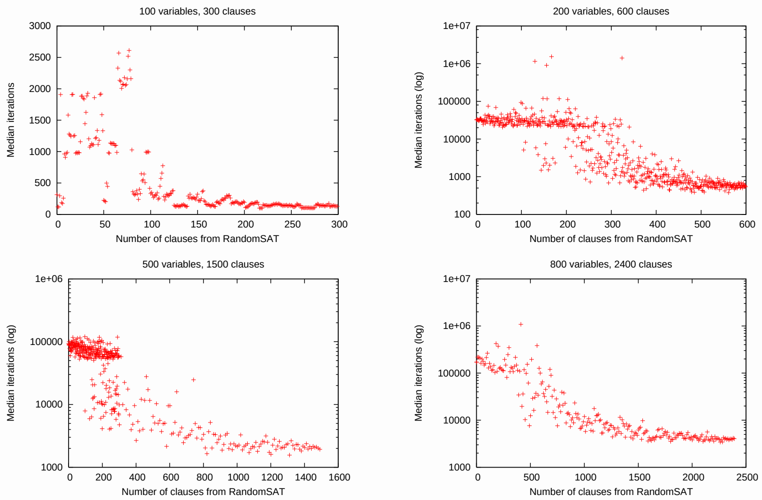
GSAT



WalkSAT



ILS-SAT



Discussion

- Many small-world/lattice SAT instances are harder for GSAT and ILS-SAT
- WalkSAT exhibits a peculiar behavior

► The relation between SATgraph and search landscape plays a very important role

Future work

- **Connections between constraint graph properties and search space characteristics**
- Exploring strengths and weaknesses of the heuristics w.r.t. constraint graph properties
- Relation between problem encoding and graph properties
- Alternative formulations to study the structure of a problem can be used (e.g., weighted graphs)