Description Logics and Ontologies: myths and challenges

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Summary

- What is an Ontology
- (Description) Logics for Conceptual Modelling
- Querying a DB via a Conceptual Schema

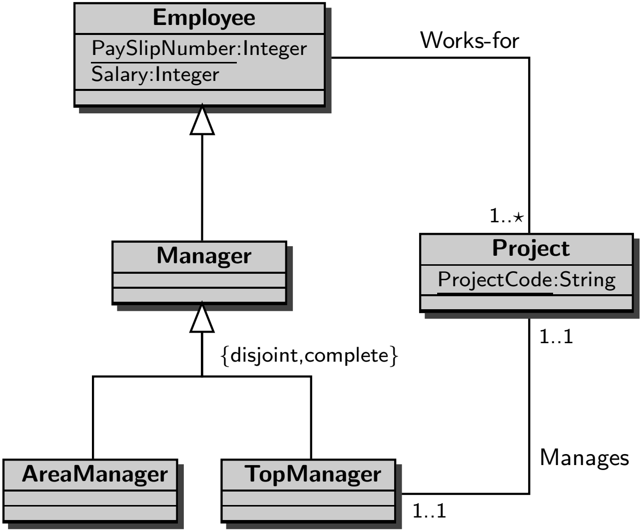
Ontologies and Constraints

- An ontology is a formal conceptualisation of the world: a conceptual schema.
- An ontology specifies a set of constraints, which declare what should necessarily hold in any possible world.
- Any possible world should conform to the constraints expressed by the ontology.
- Given an ontology, a *legal world description* (or legal database instance) is a finite possible world satisfying the constraints.

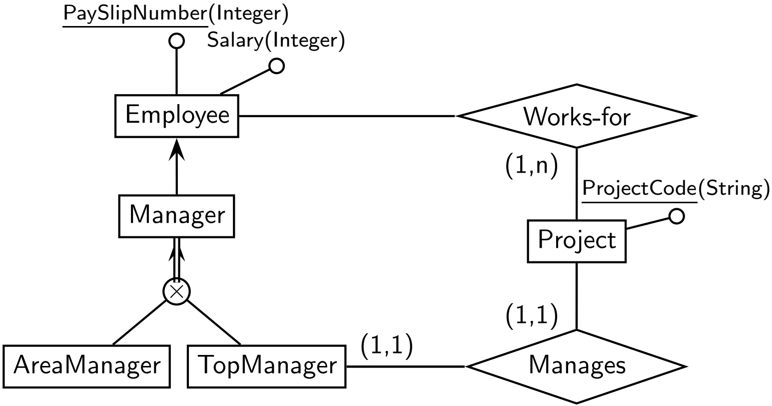
Ontologies and Conceptual Data Models

- An ontology language usually introduces concepts (aka classes, entities), properties of concepts (aka slots, attributes, roles), relationships between concepts (aka associations), and additional constraints.
- Ontology languages may be simple (e.g., involving only concepts and taxonomies), frame-based (e.g., UML, based on concepts, properties, and binary relationships), or logic-based (e.g. OWL, Description Logics).
- Ontology languages are typically expressed by means of diagrams.
- Entity-Relationship schemas and UML class diagrams can be considered as ontology languages.

UML Class Diagram



Entity-Relationship Schema



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Ontologies, Constraints, and Logic

- A database instance is a (finite) relational structure (an interpretation).
- Constraints can be seen as (first-order) logic formulas
- $\rightarrow$ satisfaction
- $\rightarrow$ querying

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- An incomplete database can be expressed by means of logic formulas
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- Incomplete specification of a world description: incomplete database as a set of database instances
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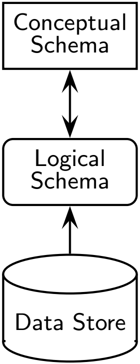
- (By the way, also a single database instance can be expressed by means of logic formulas $\rightarrow$ Reiter)

The role of a Conceptual Schema: an Ontology based application



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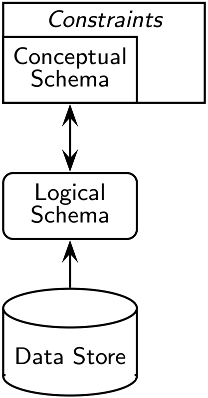

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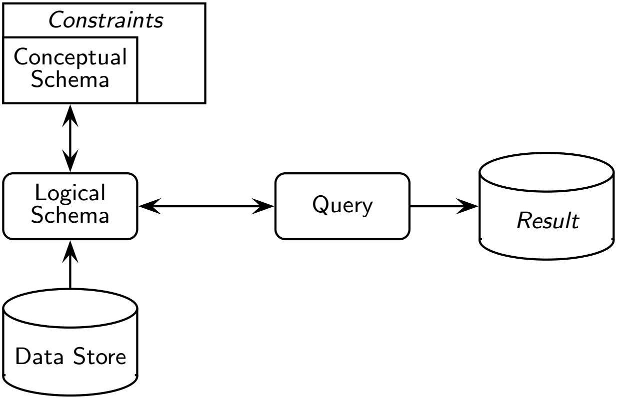

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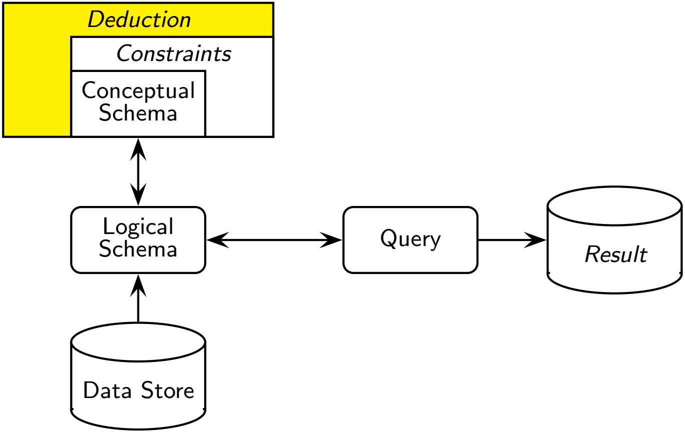

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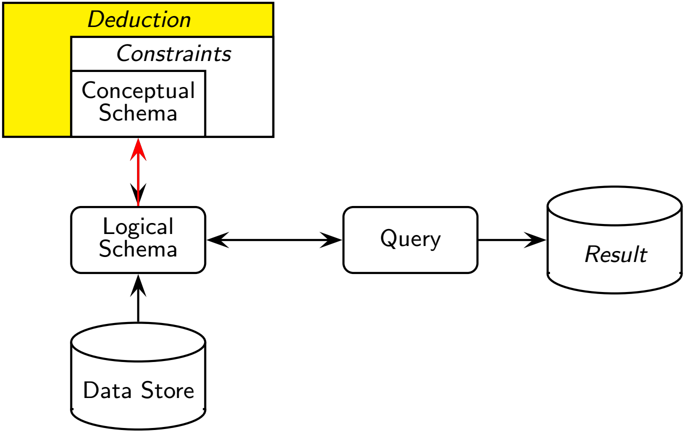

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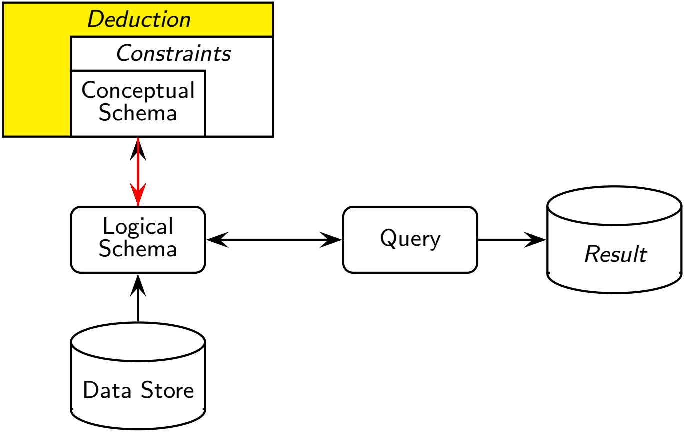

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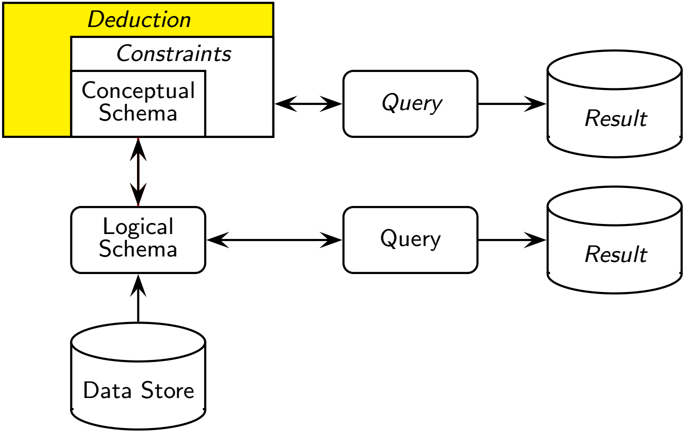

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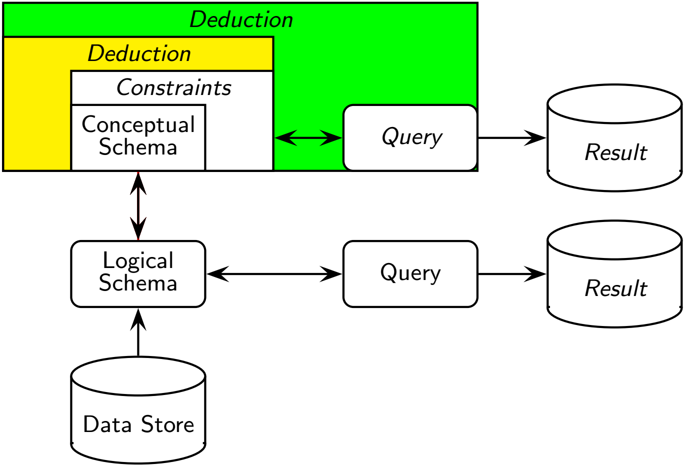

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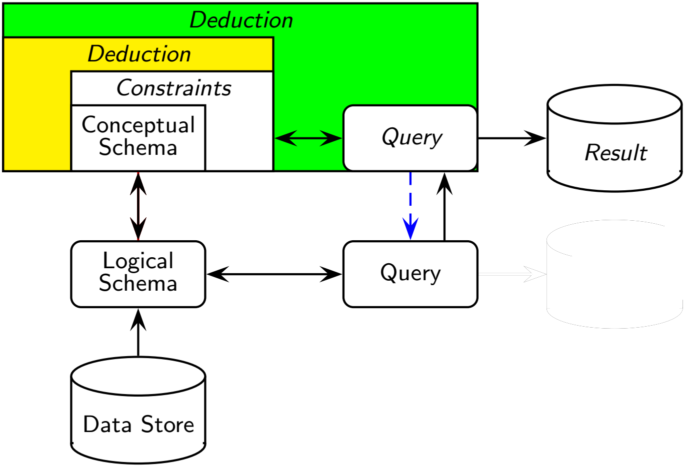

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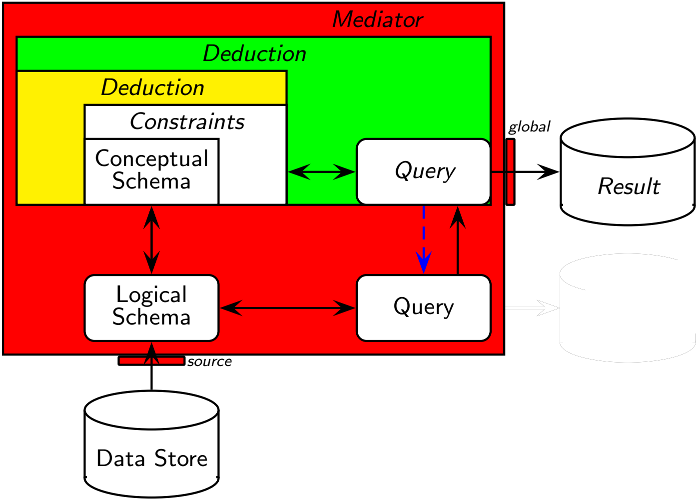

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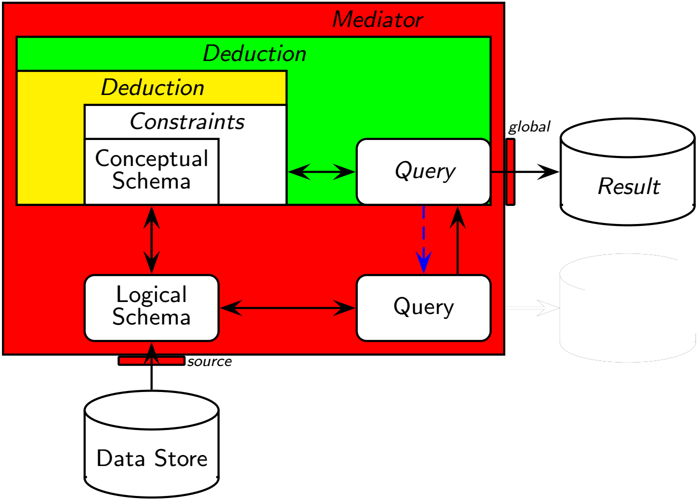


⟵ Knowledge Level

⟵ Information Level

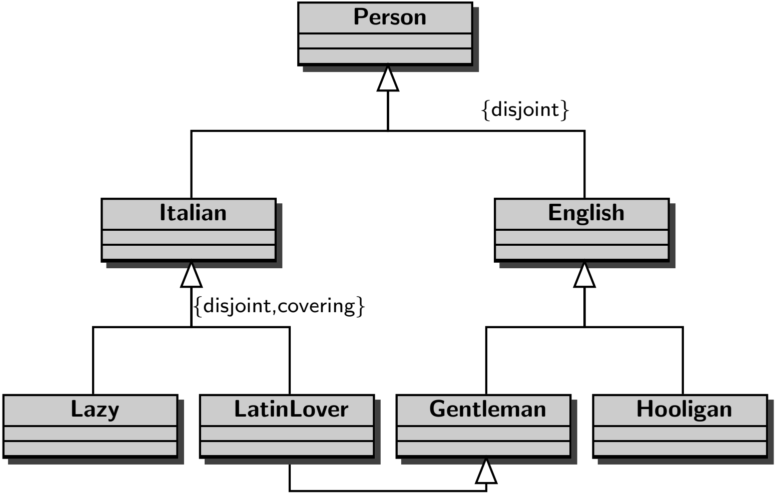
⟵ Data Level

Reasoning

Given an ontology – seen as a collection of constraints – it is possible that additional constraints can be inferred.

- A class is inconsistent if it denotes the empty set in any legal world description.
- A class is a subclass of another class if the former denotes a subset of the set denoted by the latter in any legal world description.
- Two classes are equivalent if they denote the same set in any legal world description.
- A stricter constraint is inferred – e.g., a cardinality constraint – if it holds in in any legal world description.
- ...

Simple reasoning example



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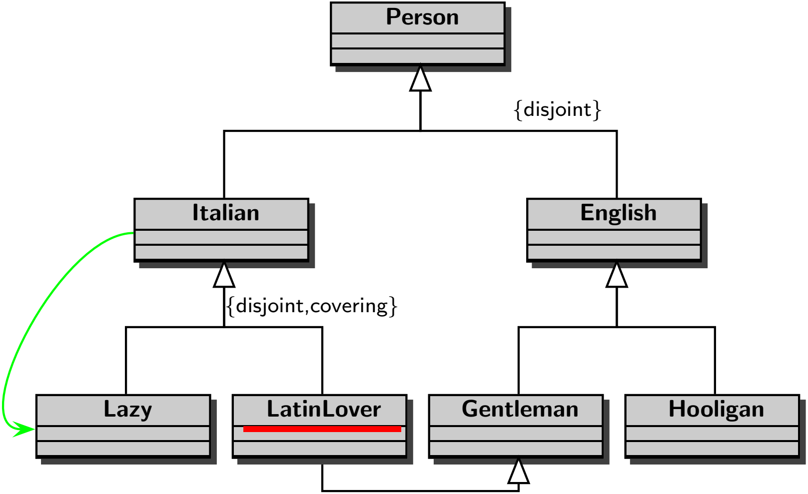
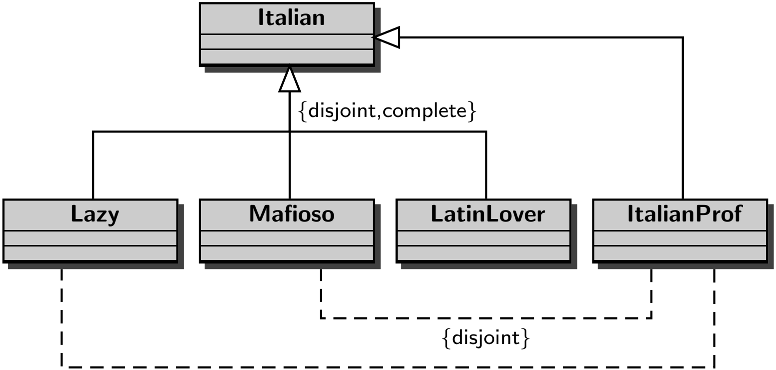


$\text{LatinLover} = \emptyset$

$\text{Italian} \subseteq \text{Lazy}$

$\text{Italian} \equiv \text{Lazy}$

Reasoning: cute professors



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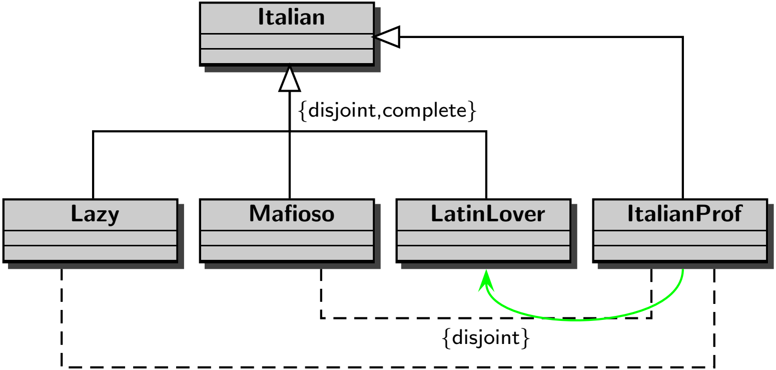
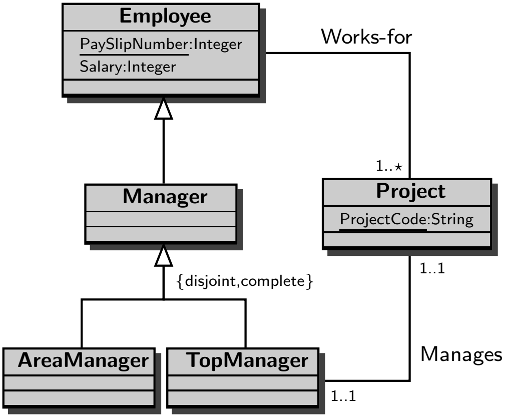


implies

ItalianProf $\subseteq$ LatinLover

Reasoning with Conceptual Schemas



- Managers do not work for a project (she/he just manages it):

  $\forall x.\, \texttt{Manager}(x) \rightarrow \neg\exists y.\, \texttt{WORKS-FOR}(x, y)$

  $\texttt{Manager} \sqsubseteq \neg\exists \texttt{WORKS-FOR}.\top$

  $\texttt{Manager} \subseteq \texttt{Employee} \setminus \pi_1 \texttt{WORKS-FOR}$

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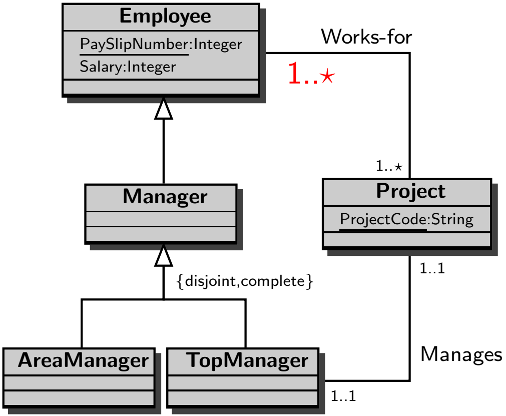

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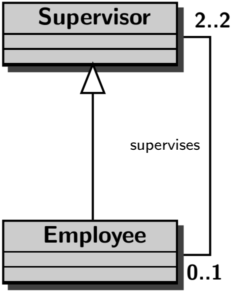
- If the minimum cardinality for the participation of employees to the *works-for* relationship is increased, then ...

The democratic company



Employee $\neq \emptyset$

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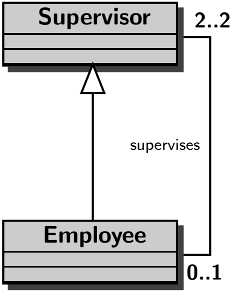
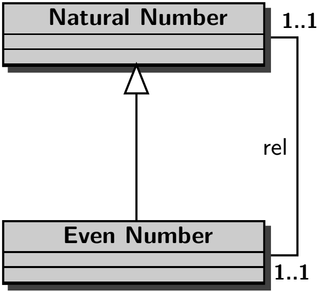


Employee $\neq \emptyset$

implies
"the classes Employee and Supervisor necessarily contain an infinite number of instances".

Since legal world descriptions are *finite* possible worlds satisfying the constraints imposed by the conceptual schema, the schema is inconsistent.

How many numbers?



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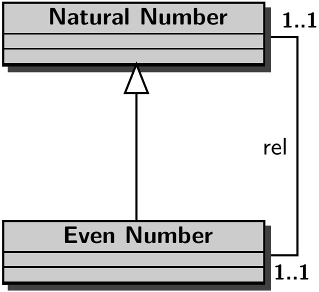
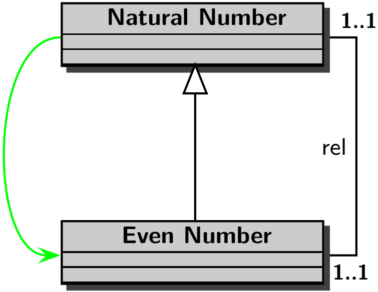


implies
"the classes Natural Number and Even Number contain the same number of instances".

How many numbers?



implies
"the classes Natural Number and Even Number contain the same number of instances".

Only if the domain is finite: Natural Number $\equiv$ Even Number

Summary

- What is an Ontology
- $\boxed{\text{(Description) Logics for Conceptual Modelling}}$
- Querying a DB via a Conceptual Schema

Encoding Conceptual Schemas in (Description) Logics

- Object-oriented data models (e.g., UML and ODMG)
- Semantic data models (e.g., EER and ORM)
- Frame-based and web ontology languages (e.g., OWL)

Encoding Conceptual Schemas in (Description) Logics

- Object-oriented data models (e.g., UML and ODMG)
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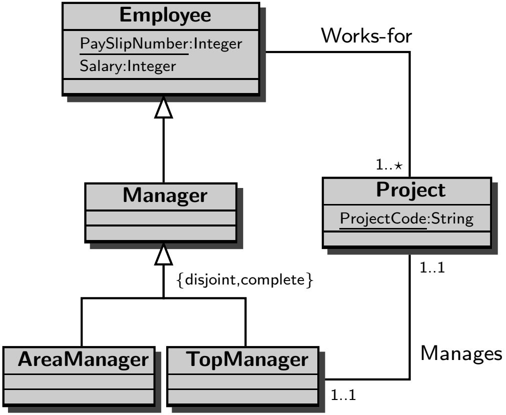
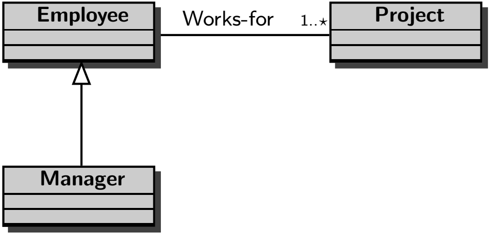
- Frame-based and web ontology languages (e.g., OWL)

- Theorems **prove** that a conceptual schema and its encoding as DL knowledge bases constrain every world description in the same way – i.e., the models of the DL theory correspond to the legal world descriptions of the conceptual schema, and vice-versa.



$$
\begin{array}{lcl}
\texttt{Works-for} & \sqsubseteq & \texttt{emp}/2 : \texttt{Employee} \sqcap \texttt{act}/2 : \texttt{Project} \\
\texttt{Manages} & \sqsubseteq & \texttt{man}/2 : \texttt{TopManager} \sqcap \texttt{prj}/2 : \texttt{Project} \\
\texttt{Employee} & \sqsubseteq & \exists^{=1}[\texttt{worker}](\texttt{PaySlipNumber} \sqcap \texttt{num}/2 : \texttt{Integer}) \sqcap \\
 & & \exists^{=1}[\texttt{payee}](\texttt{Salary} \sqcap \texttt{amount}/2 : \texttt{Integer}) \\
\top & \sqsubseteq & \exists^{\leq 1}[\texttt{num}](\texttt{PaySlipNumber} \sqcap \texttt{worker}/2 : \texttt{Employee}) \\
\texttt{Manager} & \sqsubseteq & \texttt{Employee} \sqcap (\texttt{AreaManager} \sqcup \texttt{TopManager}) \\
\texttt{AreaManager} & \sqsubseteq & \texttt{Manager} \sqcap \neg\texttt{TopManager} \\
\texttt{TopManager} & \sqsubseteq & \texttt{Manager} \sqcap \exists^{=1}[\texttt{man}]\texttt{Manages} \\
\texttt{Project} & \sqsubseteq & \exists^{\geq 1}[\texttt{act}]\texttt{Works-for} \sqcap \exists^{=1}[\texttt{prj}]\texttt{Manages} \\
\ldots & &
\end{array}
$$

Relational algebra constraints



Employee/1, Manager/1, Project/1, Works-for/2

Manager $\subseteq$ Employee

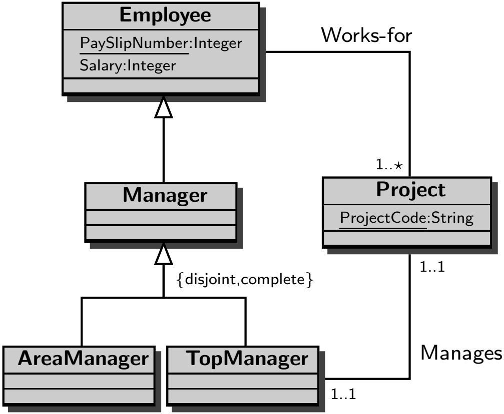
π_1 Works-for $\subseteq$ Employee
π_2 Works-for $\subseteq$ Project

Project $\subseteq$ π_2 Works-for

Set-based Constraints

$$\text{Works-for} \subseteq \text{Employee} \times \text{Project}$$
$$\text{Manages} \subseteq \text{TopManager} \times \text{Project}$$
$$\text{Employee} \subseteq \{e \mid \sharp(\text{PaySlipNumber} \cap (\{e\} \times \texttt{Integer})) \geq 1\}$$
$$\text{Employee} \subseteq \{e \mid \sharp(\text{Salary} \cap (\{e\} \times \texttt{Integer})) \geq 1\}$$
$$\text{Project} \subseteq \{p \mid \sharp(\text{ProjectCode} \cap (\{p\} \times \texttt{String})) \geq 1\}$$
$$\text{TopManager} \subseteq \{m \mid 1 \geq \sharp(\text{Manages} \cap (\{m\} \times \Omega)) \geq 1\}$$
$$\text{Project} \subseteq \{p \mid 1 \geq \sharp(\text{Manages} \cap (\Omega \times \{p\})) \geq 1\}$$
$$\text{Project} \subseteq \{p \mid \sharp(\text{Works-for} \cap (\Omega \times \{p\})) \geq 1\}$$
$$\text{Manager} \subseteq \text{Employee}$$
$$\text{AreaManager} \subseteq \text{Manager}$$
$$\text{TopManager} \subseteq \text{Manager}$$
$$\text{AreaManager} \cap \text{TopManager} = \emptyset$$
$$\text{Manager} \subseteq \text{AreaManager} \cup \text{TopManager}$$

Deducing constraints



Managers are employees who do not work for a project (she/he just manages it):

$\mathtt{Employee} \sqcap \neg(\exists^{\geq 1}[\mathtt{emp}]\mathtt{Works\text{-}for}) \sqsubseteq \mathtt{Manager}, \quad \mathtt{Manager} \sqsubseteq \neg(\exists^{\geq 1}[\mathtt{emp}]\mathtt{Works\text{-}for})$

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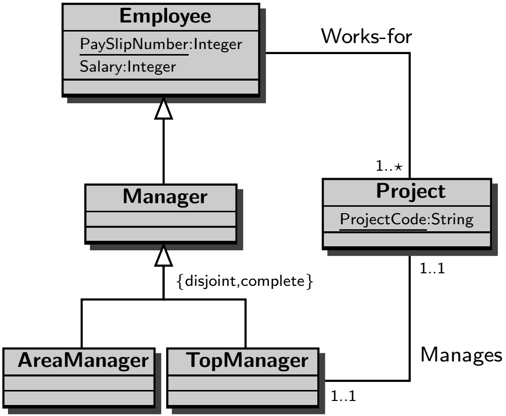

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$\models$ For every project, there is at least one employee who is not a manager:

$\texttt{Project} \sqsubseteq \exists^{\geq 1}[\texttt{act}](\texttt{Works-for} \sqcap \texttt{emp} : \neg\texttt{Manager})$

i•com: **Intelligent Conceptual Modelling**

- i•com allows for the specification of multiple EER (or UML) diagrams and inter- and intra-schema constraints;
- Complete logical reasoning is employed by the tool using a hidden underlying $\mathcal{DLR}$ inference engine;
- i•com verifies the specification, infers implicit facts and stricter constraints, and manifests any inconsistencies during the conceptual modelling phase.
- `http://www.inf.unibz.it/~franconi/icom/`

$\boxed{\text{Next}}$ on "Myths and Challenges":

- What is an Ontology
- (Description) Logics for Conceptual Modelling
- $\boxed{\text{Querying a DB via a Conceptual Schema}}$
  - We will see how an ontology can play the role of a "mediator" wrapping a (source) database.
  - Examples will show how apparently simple cases are not easy.
  - We will learn about view-based query processing with GAV and LAV mappings.
  - We introduce the difference between closed world and open world semantics in this context.
  - We will see how only the closed world semantics should be used while using ontologies to wrap databases, in order for the mediated system to behave like a database (black-box metaphor)
  - We will see that the data complexity of query answering can be beyond the one of SQL.

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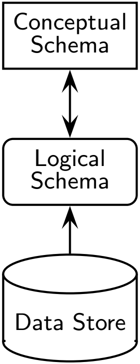
The role of a Conceptual Schema



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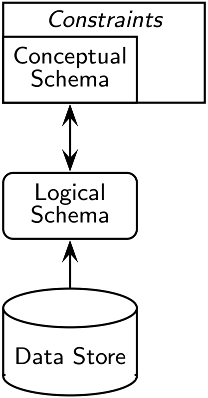

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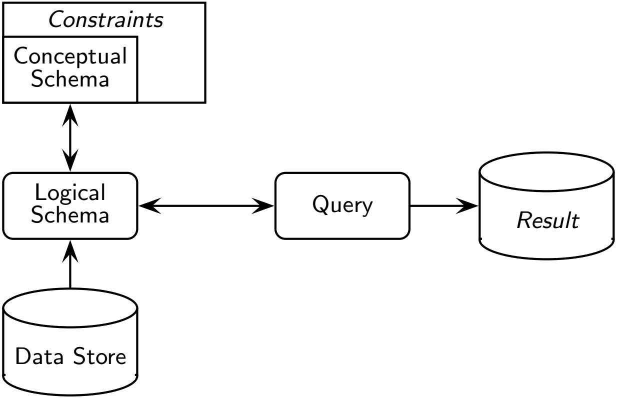

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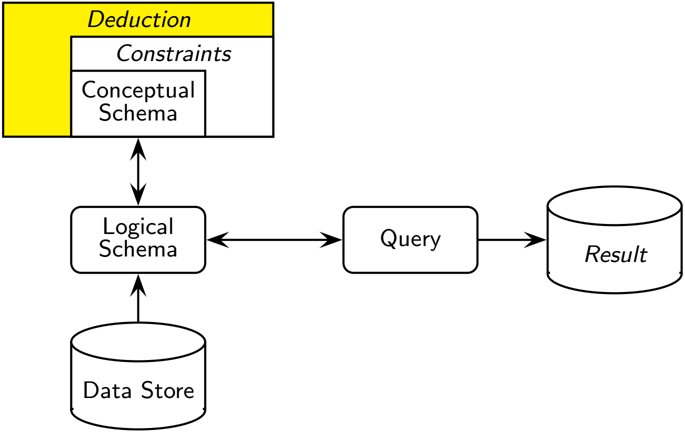

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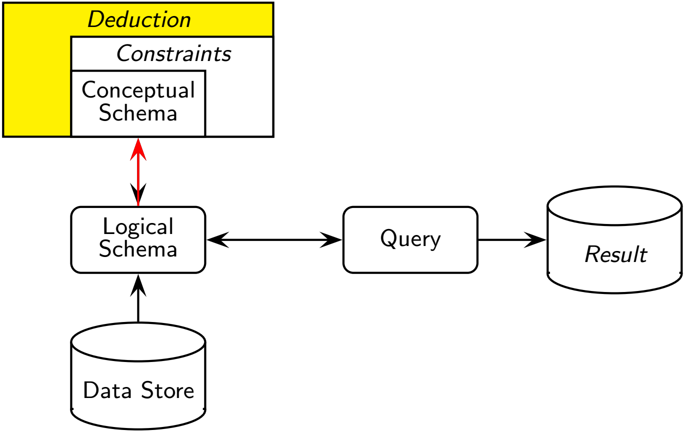

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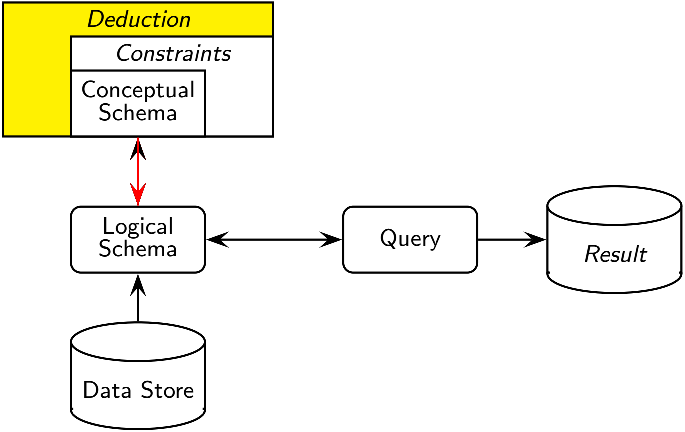

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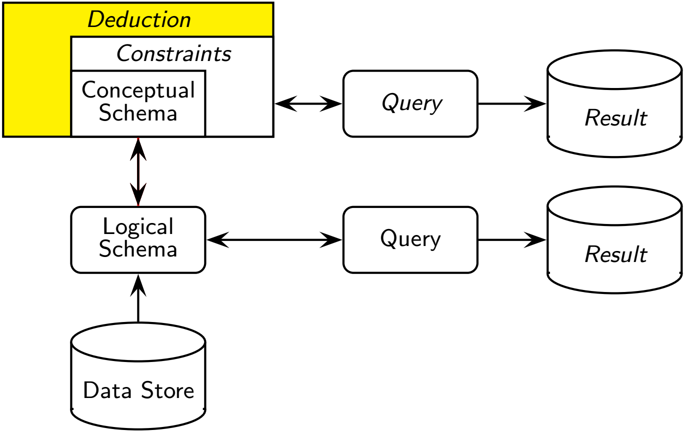

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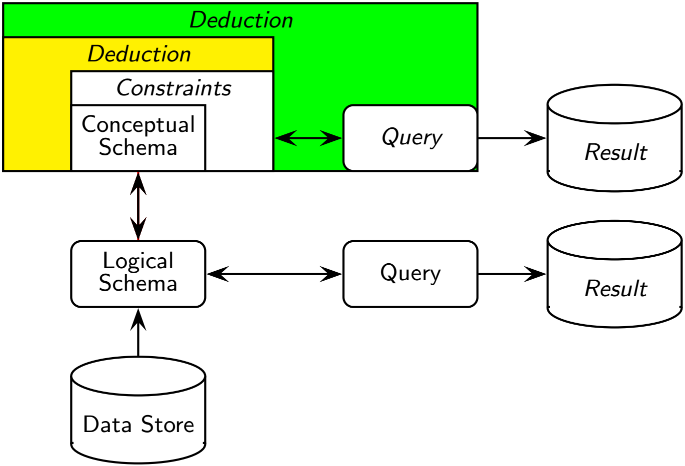

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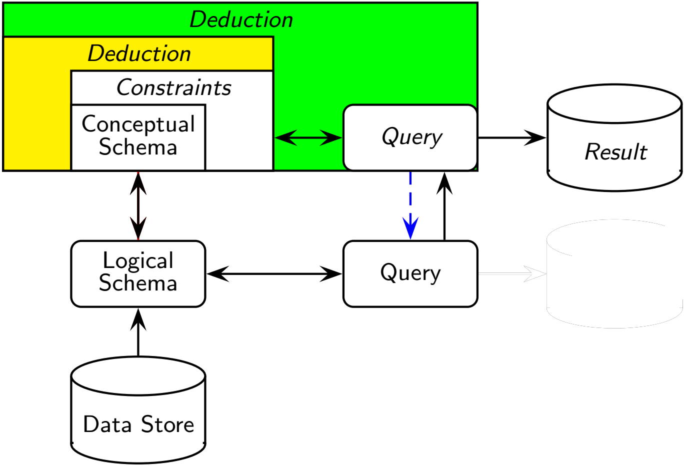

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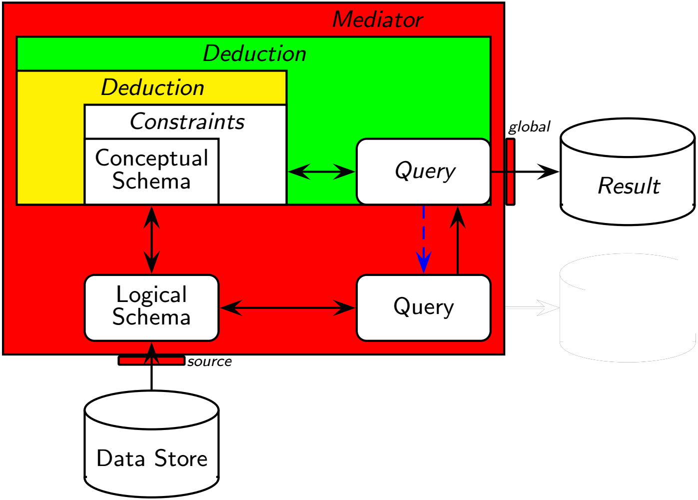

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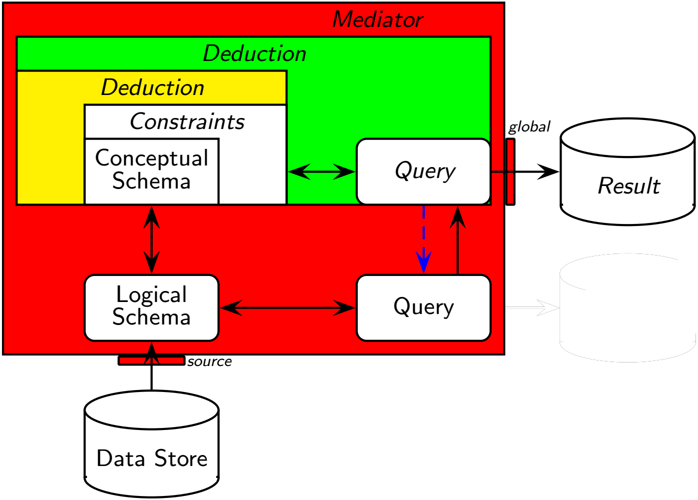

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Querying a Database with Constraints

- Basic assumption: consistent information with respect to the constraints introduced by the conceptual schema
- A Database with Constraints: complete information about each term appearing in the conceptual schema
- *Problem*: answer a query over the conceptual schema vocabulary

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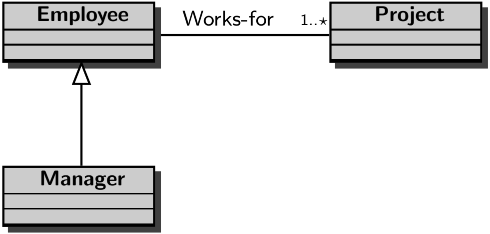
- A Database with Constraints: complete information about each term appearing in the conceptual schema
- *Problem*: answer a query over the conceptual schema vocabulary
- *Solution*: use a standard DB technology (e.g., SQL, datalog, etc)

Querying a Database with Constraints



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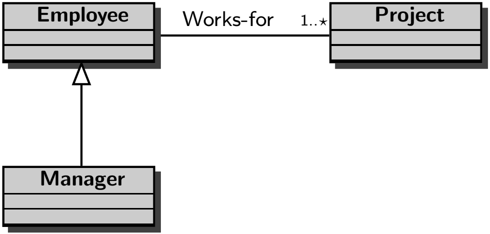

Employee = { John, Mary, Paul }
Manager = { John, Paul }
Works-for = { ⟨John,Prj-A⟩, ⟨Mary,Prj-B⟩ }
Project = { Prj-A, Prj-B }

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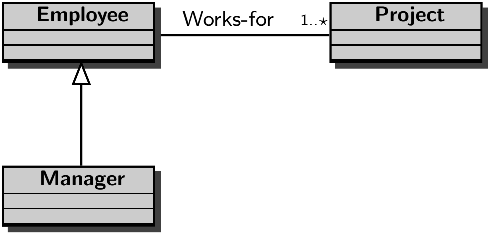

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Q(X) :- Manager(X), Works-for(X,Y), Project(Y)
$\Longrightarrow$ { John }

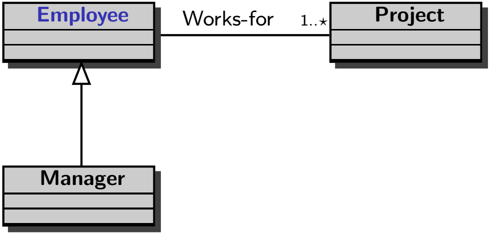
Querying a Database with Constraints over an extended signature (DBox)

- Having a classical database with constraints is against the principle that a conceptual schema presents a richer vocabulary than the data stores (i.e., it plays the role of an ontology).

Querying a Database with Constraints over an extended signature (DBox)

- Having a classical database with constraints is against the principle that a conceptual schema presents a richer vocabulary than the data stores (i.e., it plays the role of an ontology).
- A Database with Constraints over an extended signature (or conceptual schema with *exact views*, or DBox): complete information about *some* term appearing in the conceptual schema
- Standard DB technologies do not apply
- The query answering problem in this context is inherently complex

Querying a Database with Constraints over an extended signature (DBox)

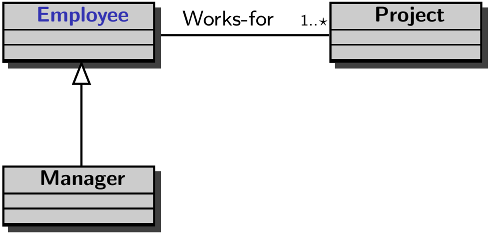


Manager = { John, Paul }

Works-for = { ⟨John,Prj-A⟩, ⟨Mary,Prj-B⟩ }

Project = { Prj-A, Prj-B }

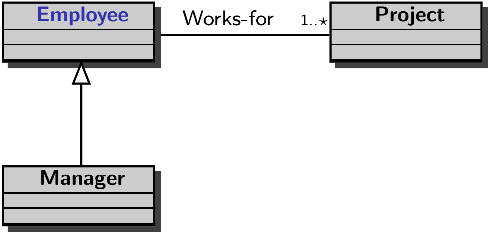
Querying a Database with Constraints over an extended signature (DBox)




```
Manager = { John, Paul }
Works-for = { ⟨John,Prj-A⟩, ⟨Mary,Prj-B⟩ }
Project = { Prj-A, Prj-B }

Q(X) :- Employee(X)
```

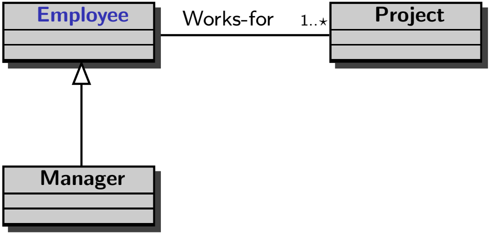
Querying a Database with Constraints over an extended signature (DBox)



Manager = { John, Paul }
Works-for = { ⟨John,Prj-A⟩, ⟨Mary,Prj-B⟩ }
Project = { Prj-A, Prj-B }

Q(X) :- Employee(X)
$\Longrightarrow$ { John, Paul, Mary }

Querying a Database with Constraints over an extended signature (DBox)

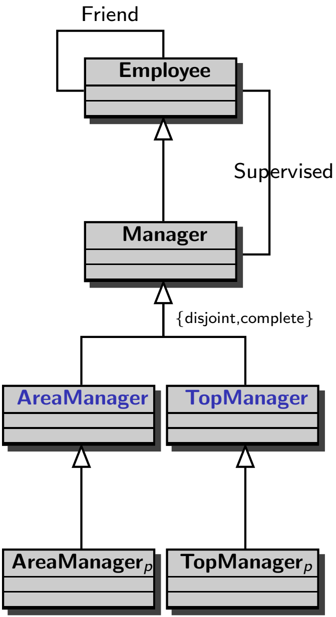


Manager = { John, Paul }
Works-for = { ⟨John,Prj-A⟩, ⟨Mary,Prj-B⟩ }
Project = { Prj-A, Prj-B }

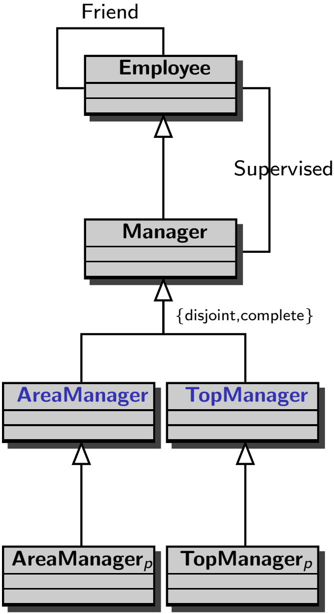
Q(X) :- Employee(X)
$\Longrightarrow$ { John, Paul, Mary }

$\Longrightarrow$ Q'(X) :- Manager(X) ∪ Works-for(X,Y)

Andrea's Example

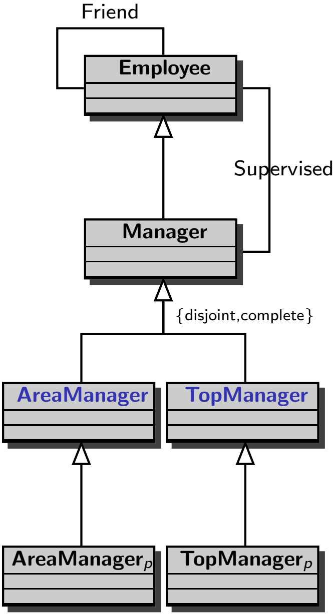


Andrea's Example



Employee = { Andrea, Paul, Mary, John }
Manager = { Andrea, Paul, Mary}
AreaManager$_p$ = { Paul }
TopManager$_p$ = { Mary }
Supervised = { ⟨John,Andrea⟩, ⟨John,Mary⟩ }
Friend = { ⟨Mary,Andrea⟩, ⟨Andrea,Paul⟩ }

Andrea's Example



$\texttt{Employee} = \{\ \texttt{Andrea},\ \texttt{Paul},\ \texttt{Mary},\ \texttt{John}\ \}$

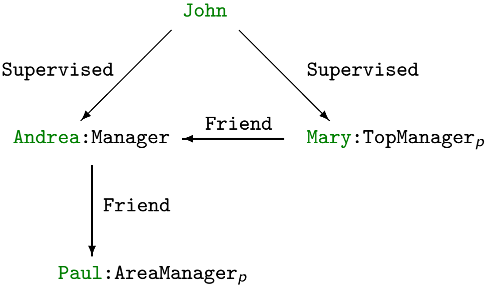
$\texttt{Manager} = \{\ \texttt{Andrea},\ \texttt{Paul},\ \texttt{Mary}\}$

$\texttt{AreaManager}_p = \{\ \texttt{Paul}\ \}$

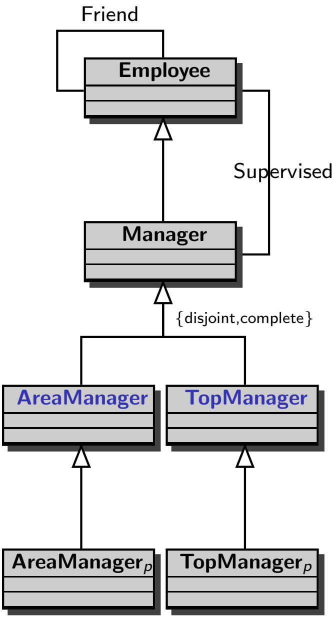
$\texttt{TopManager}_p = \{\ \texttt{Mary}\ \}$

$\texttt{Supervised} = \{\ \langle\texttt{John},\texttt{Andrea}\rangle,\ \langle\texttt{John},\texttt{Mary}\rangle\ \}$

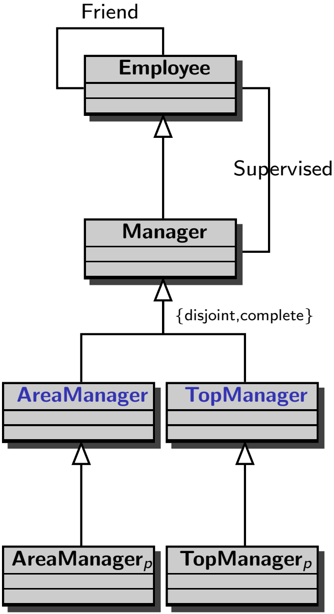
$\texttt{Friend} = \{\ \langle\texttt{Mary},\texttt{Andrea}\rangle,\ \langle\texttt{Andrea},\texttt{Paul}\rangle\ \}$

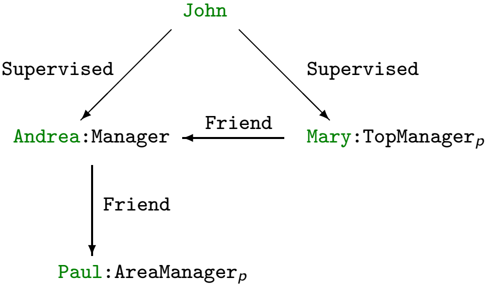


Andrea's Example (cont.)

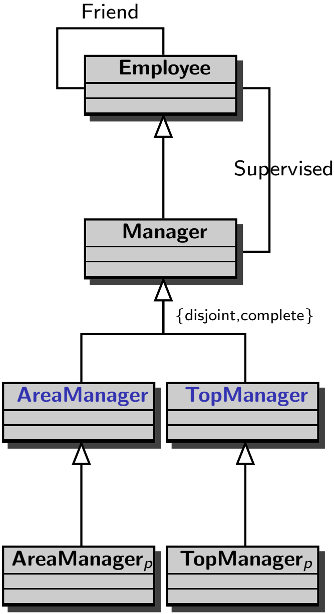


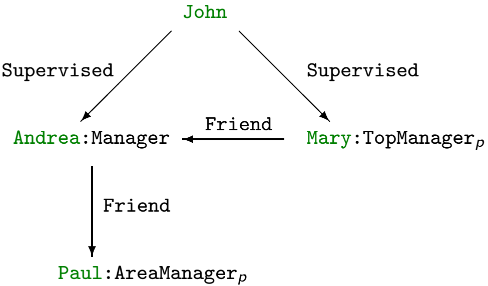
Andrea's Example (cont.)





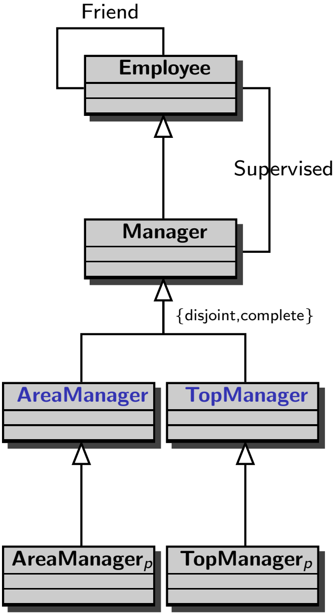
Andrea's Example (cont.)

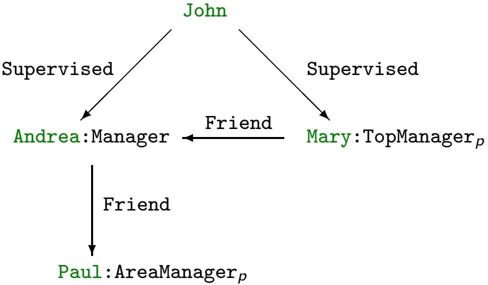





```
Q :- Supervised(John,Y), TopManager(Y),
     Friend(Y,Z), AreaManager(Z)
```

Andrea's Example (cont.)





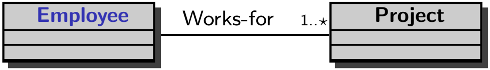

```
Q :- Supervised(John,Y), TopManager(Y),
     Friend(Y,Z), AreaManager(Z)
```

$\Longrightarrow$ YES

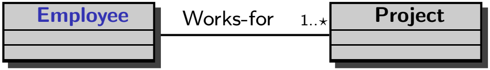
Querying a sound DB with Constraints over an extended signature (ABox)

1. Classical DB with constraints: complete information about *all* terms appearing in the conceptual schema
2. DB with constraints over an extended signature (i.e., conceptual schema with ***exact views***, or DBox): complete information about *some* term appearing in the conceptual schema
3. Sound DB with constraints over an extended signature (aka conceptual schema with ***sound views***, or ABox): incomplete information about *some* term appearing in the conceptual schema
   - Sound databases with constraints over an extended signature are crucial in data integration scenarios.

Exact vs Sound views



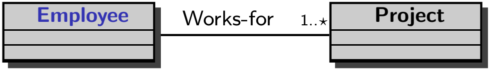
Exact vs Sound views



Exact views (DBox):

`Works-for = { ⟨John,Prj-A⟩, ⟨Mary,Prj-A⟩ }`
`Project = { Prj-A, Prj-B }`

Exact vs Sound views

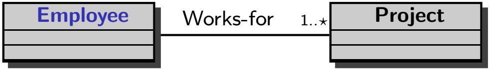


Exact views (DBox):

`Works-for = { ⟨John,Prj-A⟩, ⟨Mary,Prj-A⟩ }`
`Project = { Prj-A, Prj-B }`

$\Longrightarrow$ INCONSISTENT

Exact vs Sound views



Exact views (DBox):

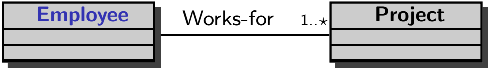
$\texttt{Works-for} = \{ \langle \texttt{John},\texttt{Prj-A} \rangle, \langle \texttt{Mary},\texttt{Prj-A} \rangle \}$
$\texttt{Project} = \{ \texttt{Prj-A}, \texttt{Prj-B} \}$

$\Longrightarrow$ INCONSISTENT

Sound views (ABox):

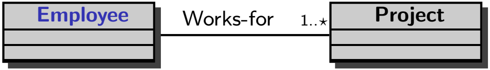
$\texttt{Works-for} \supseteq \{ \langle \texttt{John},\texttt{Prj-A} \rangle, \langle \texttt{Mary},\texttt{Prj-A} \rangle \}$
$\texttt{Project} \supseteq \{ \texttt{Prj-A}, \texttt{Prj-B} \}$

Querying a sound DB with Constraints over an extended signature (ABox)



$\texttt{Works-for} \supseteq \{ \langle \texttt{John}, \texttt{Prj-A} \rangle, \langle \texttt{Mary}, \texttt{Prj-A} \rangle \}$
$\texttt{Project} \supseteq \{ \texttt{Prj-A}, \texttt{Prj-B} \}$

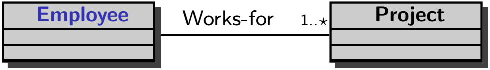
Querying a sound DB with Constraints over an extended signature (ABox)



$\texttt{Works-for} \supseteq \{ \langle \texttt{John},\texttt{Prj-A}\rangle, \langle \texttt{Mary},\texttt{Prj-A}\rangle \}$
$\texttt{Project} \supseteq \{ \texttt{Prj-A}, \texttt{Prj-B} \}$

`Q(X) :- Works-for(Y,X)`

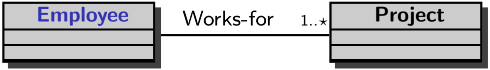
Querying a sound DB with Constraints over an extended signature (ABox)



$\texttt{Works-for} \supseteq \{ \langle \texttt{John}, \texttt{Prj-A} \rangle, \langle \texttt{Mary}, \texttt{Prj-A} \rangle \}$
$\texttt{Project} \supseteq \{ \texttt{Prj-A}, \texttt{Prj-B} \}$

`Q(X) :- Works-for(Y,X)`
$\Longrightarrow \{ \texttt{Prj-A}, \texttt{Prj-B} \}$

Querying a sound DB with Constraints over an extended signature (ABox)

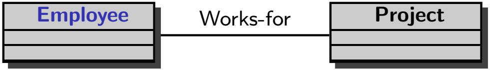


$\texttt{Works-for} \supseteq \{\ \langle \texttt{John}, \texttt{Prj-A} \rangle,\ \langle \texttt{Mary}, \texttt{Prj-A} \rangle\ \}$
$\texttt{Project} \supseteq \{\ \texttt{Prj-A},\ \texttt{Prj-B}\ \}$

$\texttt{Q(X) :- Works-for(Y,X)}$
$\Longrightarrow \{\ \texttt{Prj-A},\ \texttt{Prj-B}\ \}$

$\Longrightarrow \qquad \texttt{Q'(X) :- Project(X)} \cup \texttt{Works-for(Y,X)}$

DBox vs ABox



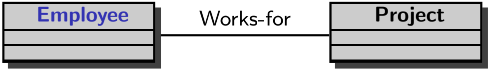
- Additional constraint as a standard view over the data:

$\texttt{Bad-Project} = \texttt{Project} \setminus \pi_2\texttt{Works-for}$

$\forall \texttt{x}.\ \texttt{Bad-Project(x)} \leftrightarrow \texttt{Project(x)} \land \neg \exists \texttt{y}.\texttt{Works-for(y,x)}$

$\texttt{Bad-Project} = \texttt{Project} \sqcap \neg \exists \texttt{Works-for}^{-}.\top$

DBox vs ABox



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  $\texttt{Bad-Project} = \texttt{Project} \sqcap \neg \exists \texttt{Works-for}^{-}.\top$
- DBox:
  $\texttt{Works-for} = \{ \langle \texttt{John},\texttt{Prj-A} \rangle, \langle \texttt{Mary},\texttt{Prj-A} \rangle \}$
  $\texttt{Project} = \{ \texttt{Prj-A}, \texttt{Prj-B} \}$
- `Q(X) :- Bad-Project(X)`

- ABox:
  $\texttt{Works-for} \supseteq \{ \langle \texttt{John},\texttt{Prj-A} \rangle, \langle \texttt{Mary},\texttt{Prj-A} \rangle \}$
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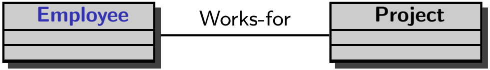
DBox vs ABox



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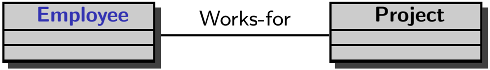
  $\texttt{Bad-Project} = \texttt{Project} \sqcap \neg \exists \texttt{Works-for}^{-}.\top$
- DBox:
  $\texttt{Works-for} = \{\ \langle \texttt{John},\texttt{Prj-A} \rangle,\ \langle \texttt{Mary},\texttt{Prj-A} \rangle\ \}$
  $\texttt{Project} = \{\ \texttt{Prj-A},\ \texttt{Prj-B}\ \}$
- Q(X) :- Bad-Project(X)
  $\Longrightarrow \{\ \texttt{Prj-B}\ \}$
- ABox:
  $\texttt{Works-for} \supseteq \{\ \langle \texttt{John},\texttt{Prj-A} \rangle,\ \langle \texttt{Mary},\texttt{Prj-A} \rangle\ \}$
  $\texttt{Project} \supseteq \{\ \texttt{Prj-A},\ \texttt{Prj-B}\ \}$
- Q(X) :- Bad-Project(X)

DBox vs ABox



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  $\forall \texttt{x}.\ \texttt{Bad-Project(x)} \leftrightarrow \texttt{Project(x)} \wedge \neg \exists \texttt{y}.\texttt{Works-for(y,x)}$

  $\texttt{Bad-Project} = \texttt{Project} \sqcap \neg \exists \texttt{Works-for}^{-}.\top$

- DBox:

  $\texttt{Works-for} = \{ \langle \texttt{John},\texttt{Prj-A} \rangle, \langle \texttt{Mary},\texttt{Prj-A} \rangle \}$

  $\texttt{Project} = \{ \texttt{Prj-A}, \texttt{Prj-B} \}$

- `Q(X) :- Bad-Project(X)`

  $\Longrightarrow \{ \texttt{Prj-B} \}$

- ABox:

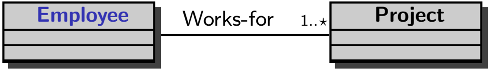
  $\texttt{Works-for} \supseteq \{ \langle \texttt{John},\texttt{Prj-A} \rangle, \langle \texttt{Mary},\texttt{Prj-A} \rangle \}$

  $\texttt{Project} \supseteq \{ \texttt{Prj-A}, \texttt{Prj-B} \}$

- `Q(X) :- Bad-Project(X)`

  $\Longrightarrow \{ \}$ *does not scale down to standard DB answer!*

Compositionality of Queries

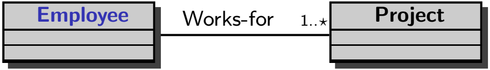


- ABox:

  $\texttt{Works-for} \supseteq \{\ \langle \texttt{John}, \texttt{Prj-A} \rangle\ \}$

  $\texttt{Project} \supseteq \{\ \texttt{Prj-A},\ \texttt{Prj-B}\ \}$

Compositionality of Queries



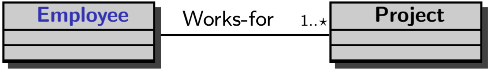
- ABox:

  $\texttt{Works-for} \supseteq \{ \langle \texttt{John}, \texttt{Prj-A} \rangle \}$
  $\texttt{Project} \supseteq \{ \texttt{Prj-A}, \texttt{Prj-B} \}$

- Query as a standard view over the data:

  `Q(X) :- Works-for(Y,X)` $\texttt{Q} = \pi_2 \texttt{Works-for}$

Compositionality of Queries



- ABox:

  $\texttt{Works-for} \supseteq \{ \langle \texttt{John}, \texttt{Prj-A} \rangle \}$
  $\texttt{Project} \supseteq \{ \texttt{Prj-A}, \texttt{Prj-B} \}$

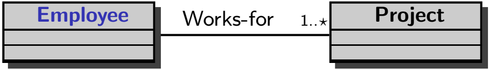
- Query as a standard view over the data:

  `Q(X) :- Works-for(Y,X)` $\qquad \texttt{Q} = \pi_2 \texttt{Works-for}$

  - $\texttt{Q} = \texttt{EVAL}(\pi_2 \texttt{Works-for})$

  - $\texttt{Q} = \pi_2(\texttt{EVAL}(\texttt{Works-for}))$

Compositionality of Queries



- ABox:

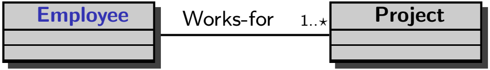
  Works-for $\supseteq$ { ⟨John,Prj-A⟩ }
  Project $\supseteq$ { Prj-A, Prj-B }

- Query as a standard view over the data:

  Q(X) :- Works-for(Y,X)   Q $= \pi_2$Works-for

  - Q $=$ EVAL(π_2Works-for)
    $\Longrightarrow$ { Prj-A, Prj-B }
  - Q $= \pi_2$(EVAL(Works-for))

Compositionality of Queries



- ABox:

  $\texttt{Works-for} \supseteq \{\ \langle\texttt{John},\texttt{Prj-A}\rangle\ \}$
  $\texttt{Project} \supseteq \{\ \texttt{Prj-A},\ \texttt{Prj-B}\ \}$

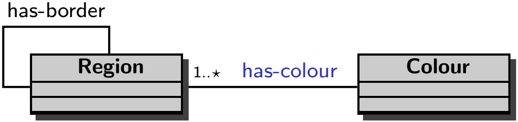
- Query as a standard view over the data:

  $\texttt{Q(X) :- Works-for(Y,X)} \qquad \texttt{Q} = \pi_2\texttt{Works-for}$

  - $\texttt{Q} = \texttt{EVAL}(\pi_2\texttt{Works-for})$
    $\Longrightarrow \{\ \texttt{Prj-A},\ \texttt{Prj-B}\ \}$
  - $\texttt{Q} = \pi_2(\texttt{EVAL}(\texttt{Works-for}))$
    $\Longrightarrow \{\ \texttt{Prj-A}\ \}$

Queries are not compositional wrt certain answer semantics!

Complexity of Query answering

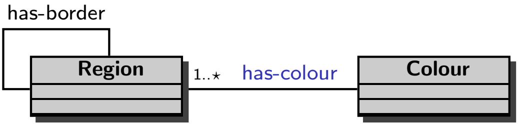


- DBox:

  $\texttt{Region} = \{\texttt{Italy},\texttt{France},\ldots\}$; $\texttt{has-border} = \{\langle\texttt{Italy},\texttt{France}\rangle,\ldots\}$;

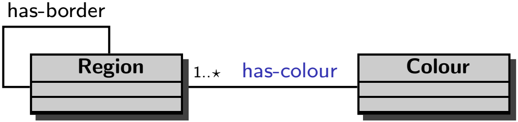
  $\texttt{Colour} = \{\ \texttt{Red},\ \texttt{Green},\ \texttt{Blue}\ \}$

Complexity of Query answering



- DBox:
  `Region` = {`Italy`,`France`,...}; `has-border` = {⟨`Italy`,`France`⟩,...};
  `Colour` = { `Red`, `Green`, `Blue` }
- `Q :- has-colour(R1,C), has-colour(R2,C), has-border(R1,R2)`
  Is it unavoidable that there are two adjacent regions with the same colour?

Complexity of Query answering



- DBox:

  Region = {Italy,France,...}; has-border = {⟨Italy,France⟩,...};

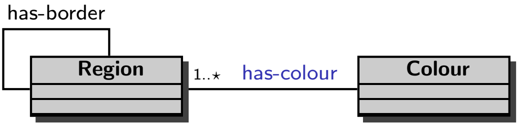
  Colour = { Red, Green, Blue }

- Q :- has-colour(R1,C), has-colour(R2,C), has-border(R1,R2)

  Is it unavoidable that there are two adjacent regions with the same colour?

  - YES: in any legal database (i.e., an assignment of colours to regions) there are at least two adjacent regions with the same colour.

Complexity of Query answering

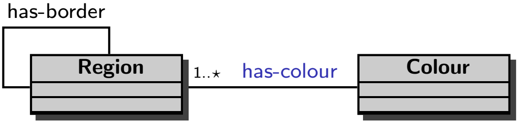


- DBox:

  Region = {Italy,France,...}; has-border = {⟨Italy,France⟩,...};
  Colour = { Red, Green, Blue }
- Q :- has-colour(R1,C), has-colour(R2,C), has-border(R1,R2)

  Is it unavoidable that there are two adjacent regions with the same colour?
  - YES: in any legal database (i.e., an assignment of colours to regions) there are at least two adjacent regions with the same colour.
  - NO: there is at least a legal database (i.e., an assignment of colours to regions) in which no two adjacent regions have the same colour.

Complexity of Query answering

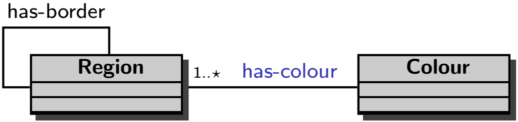


- DBox:

  Region = {Italy,France,...}; has-border = {⟨Italy,France⟩,...};
  Colour = { Red, Green, Blue }
- Q :- has-colour(R1,C), has-colour(R2,C), has-border(R1,R2)

  Is it unavoidable that there are two adjacent regions with the same colour?
  - YES: in any legal database (i.e., an assignment of colours to regions) there are at least two adjacent regions with the same colour.
  - NO: there is at least a legal database (i.e., an assignment of colours to regions) in which no two adjacent regions have the same colour.
  - With *ABox semantics* the answer is always NO, since there is at least a legal database (i.e., an assignment of colours to regions) with *enough* distinct colours so that no two adjacent regions have the same colour.

Complexity of Query answering



- DBox:

  `Region` $=$ {`Italy`,`France`,...}; `has-border` $=$ {⟨`Italy`,`France`⟩,...};

  `Colour` $=$ { `Red`, `Green`, `Blue` }

- `Q :- has-colour(R1,C), has-colour(R2,C), has-border(R1,R2)`

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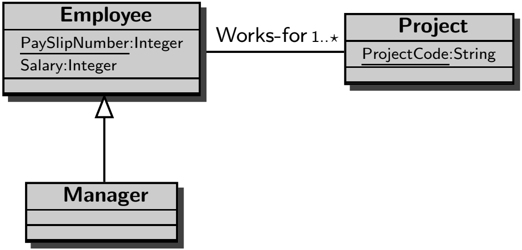
Query answering with DBoxes is co-np-hard in data complexity (3-col), and it is strictly harder than with ABoxes!

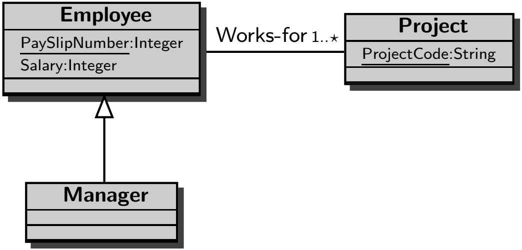
View based Query Processing

- Mappings between the conceptual schema terms and the information source terms are not necessarily atomic.
- Mappings can be given in terms of a set of sound (or exact) views:
  - GAV (*global-as-view*): sound (or exact) views over the information source vocabulary are associated to terms in the conceptual schema
    - both the DB and the partial DB assumptions are special cases of GAV
    - an ER schema can be easily mapped to its corresponding relational schema in some normal form via a GAV mapping
  - LAV (*local-as-view*): a sound or exact view over the conceptual schema vocabulary is associated to each term in the information source;
  - GLAV: mix of the above.
- It is non-trivial, even in the pure GAV setting - which is wrongly believed to be computable by simple view unfolding.
- It is mostly studied with sound views, due to the negative complexity results with exact views discussed before.

Sound GAV mapping



Sound GAV mapping




```
1-Employee(PaySlipNumber,Salary,ManagerP)
2-Works-for(PaySlipNumber,ProjectCode)
```

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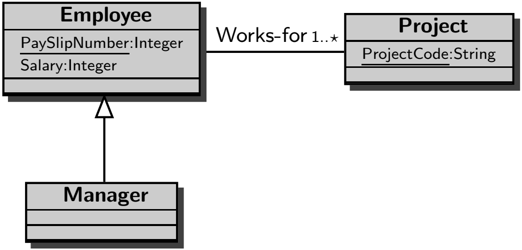


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```
Employee(X)  :-  1-Employee(X,Y,false)      Works-for(X,Y)  :-  2-Works-for(X,Y)
 Manager(X)  :-  1-Employee(X,Y,true)          Salary(X,Y)  :-  1-Employee(X,Y,Z)
 Project(Y)  :-  2-Works-for(X,Y)
```

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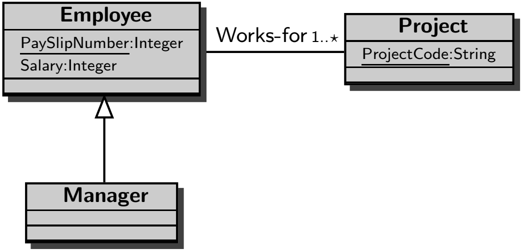


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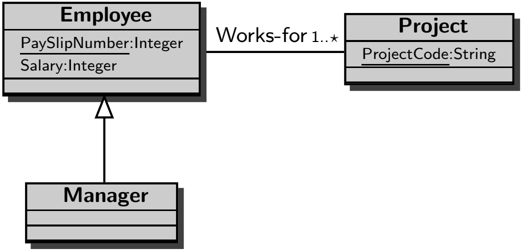


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```
Q(X) :- Employee(X)
⟹        Q'(X) :- 1-Employee(X,Y,Z) ∪ 2-Works-for(X,W)
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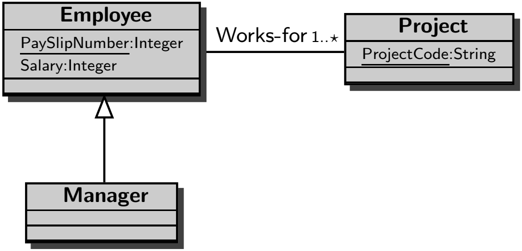

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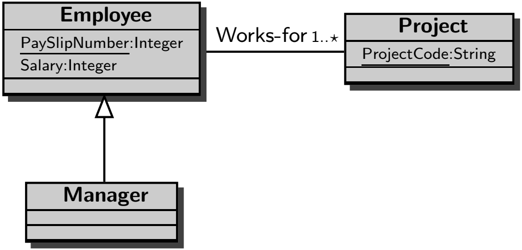
Salary(X,Y) :- 1-Employee(X,Y,Z)

Q(X) :- Employee(X)
$\Longrightarrow$ Q'(X) :- 1-Employee(X,Y,Z) $\cup$ 2-Works-for(X,W) $\leftarrow$ not coming from unfolding!

Sound LAV mapping



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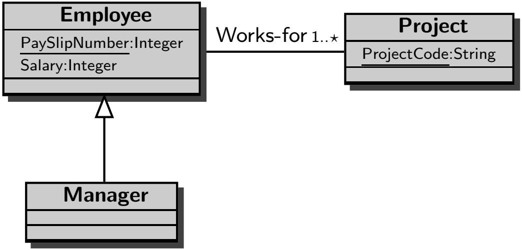

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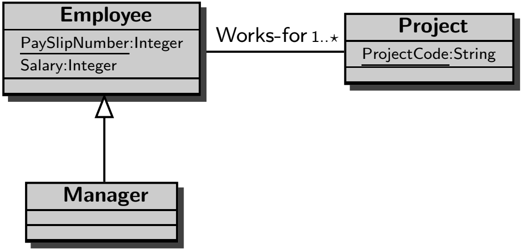

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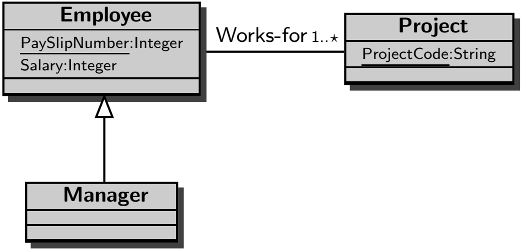

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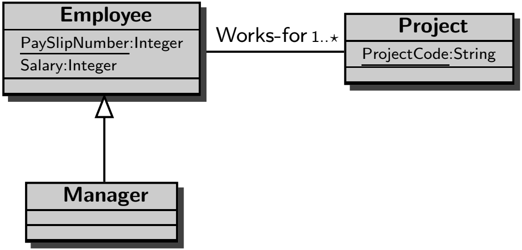


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Q(X) :- Manager(X), Works-for(X,Y), Project(Y)
⟹          Q'(X) :- 1-Employee(X,Y,true), 2-Works-for(X,Z)
```
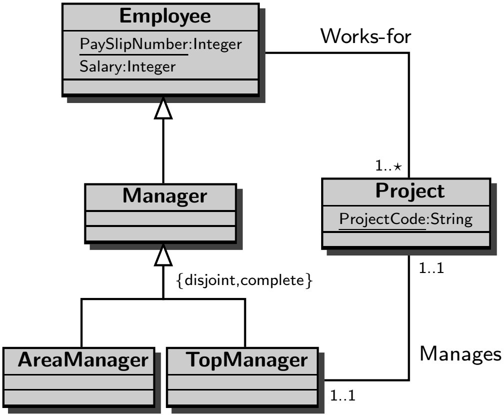

Reasoning over queries

`Q(X,Y) :- Employee(X), Works-for(X,Y), Manages(X,Y)`



$\forall x.\, \texttt{Manager}(x) \rightarrow \neg\exists y.\, \texttt{WORKS-FOR}(x, y)$

$\texttt{Manager} \sqsubseteq \neg\exists \texttt{WORKS-FOR}.\top$

$\texttt{Manager} \subseteq \texttt{Employee} \setminus \pi_1 \texttt{WORKS-FOR}$

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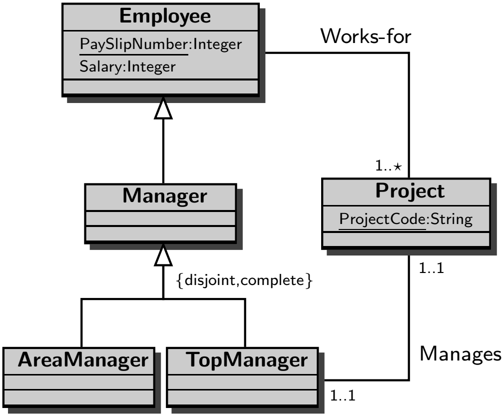

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$\mathtt{Manager} \sqsubseteq \neg\exists \mathtt{WORKS\text{-}FOR}.\top$

$\mathtt{Manager} \subseteq \mathtt{Employee} \setminus \pi_1 \mathtt{WORKS\text{-}FOR}$

$\leadsto$ INCONSISTENT QUERY!

Summary

- Logic and Conceptual Modelling
- Description Logics for Conceptual Modelling
- Queries with an Ontology
- $\boxed{\text{Determinacy}}$

Determinacy (implicit definability)

A query Q over a DBox is implicitly definable under constraints if its extension is fully determined by the extension of the DBox relations, and it does not depend on the non-DBox relations appearing in the constraints.

Checking implicit definability under first-order logic constraints of a query over a DBox can be reduced to classical entailment.

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Checking implicit definability under first-order logic constraints of a query over a DBox can be reduced to classical entailment.

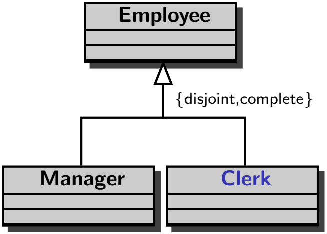
Definition (Implicit definability)

Let $\mathcal{DB}_i$ and $\mathcal{DB}_j$ be any two legal databases of the constraints $\mathcal{T}$ which agree on the extension of the DBox relations.
A query Q is *implicitly definable* from the DBox relations under the constraints $\mathcal{T}$ *iff* the answer of Q over $\mathcal{DB}_i$ is the same as the answer of Q over $\mathcal{DB}_j$.

Rewriting - or explicit definability

- If a query is implicitly definable, it is possible to find an equivalent reformulation of the query using only relations in the DBox. This is its explicit definition.
- It has been shown that under general first-order logic constraints, whenever a query is implicitly definable then it is explicitly definable in a constructive way as a first-order query.

Example



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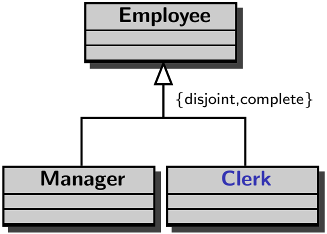

- $Q(x)$:- `Clerk`(x) is determined by the extension of the DBox relations under the constraints

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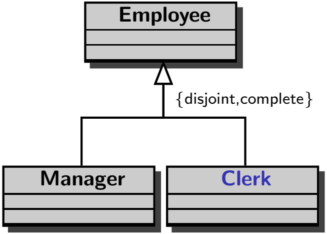

- $Q(x)$:- $\texttt{Clerk}(x)$ is determined by the extension of the DBox relations under the constraints
- $Q(x)$:- $\texttt{Clerk}(x)$ is equivalent to $Q'(x)$:- $\texttt{Employee}(x) \land \neg\texttt{Manager}(x)$

The query rewriting under constraints process

1. Check whether the database is consistent with respect to the constraints and, if so,
2. check whether the answer to the original query under first-order constraints is *solely* determined by the extension of the DBox relations and, if so,
3. find an equivalent (first-order) rewriting of the query in terms of the DBox relation.
4. It is possible to pre-compute all the rewritings of all the determined relations as SQL relational views, and to allow arbitrary SQL queries on top of them: the whole system is deployed at run time as a standard SQL relational database.

Domain independence & range-restricted rewritings

I cheated so far! ☺

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Unless the rewriting is a domain independent (e.g., a range-restricted) first-order logic formula, it can not be expressed in relational algebra or SQL!

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I cheated so far! ☺

Unless the rewriting is a domain independent (e.g., a range-restricted) first-order logic formula, it can not be expressed in relational algebra or SQL!

- We prove general conditions on the constraints and the query in order to guarantee that the rewriting is domain independent
- All the typical database constraints (e.g., TGDs and EGDs) satisfy those conditions
- All the ontology languages in the guarded fragment satisfy those conditions

Conclusions

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Do you want to exploit conceptual schema knowledge
(i.e., constraints or an ontology)
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Pay attention!