Characterizing defeat graphs where argumentation semantics agree

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Abstract. In the context of Dung's theory of argumentation frameworks, comparisons between argumentation semantics are often focused on the different behavior they show in some (more or less peculiar) cases. It is also interesting however to characterize situations where (under some reasonably general assumptions) different semantics behave exactly in the same way. Focusing on the general family of SCC-recursive argumentation semantics, the paper provides some novel results in this line. In particular, we study the characterization of defeat graphs where any SCC-recursive semantics admits exactly one extension coinciding with the grounded extension. Then, we consider the problem of agreement with stable semantics and identify the family of SCC-symmetric argumentation frameworks, where agreement is ensured for a class of multiple-status argumentation semantics including stable, preferred and $CF2$ semantics.

Key words: Argumentation semantics, Argumentation frameworks, Semantics comparison

1 Introduction

Interest in comparing argumentation semantics arises from the increasing variety of approaches proposed in the context of Dung's theory of argumentation frameworks [1]. Different behaviors exhibited by alternative semantics in specific cases (or families of cases) have often been the subject of detailed analyses and discussions about the "most intuitive" or "desired" outcome. While this is, by far, the most common kind of comparison found in the literature, a more systematic approach considering general principles that may or may not be satisfied by a semantics has also been addressed [2, 3].

A complementary kind of analysis concerns identifying situations where argumentation semantics agree, i.e. exhibit the same behavior in spite of their differences. This can be useful from several viewpoints. On one hand, situations where "most" (or even all) existing semantics agree can be regarded as providing a sort of reference behavior against which further proposals should be confronted. On the other hand, it may be the case that in a specific application domain there are some restrictions on the structure of the argumentation frameworks that need to be considered. It is then surely interesting to know whether these restrictions lead to semantics agreement, since in this case it is clear that evaluations about arguments in that domain may not be affected by different choices of argumentation semantics and are, in a sense, universally supported.

In fact, the question of semantics agreement for particular classes of argumentation frameworks is explicitly considered in Dung's original paper [1] where sufficient conditions for agreement between grounded, preferred and stable semantics and between preferred and stable semantics are provided (these results will be recalled along the paper). More recently, the special class of symmetric argumentation frameworks [4] (where every attack is mutual) has been shown to ensure agreement between preferred, stable and naive semantics. The present paper provides some new results in this area by considering the recently introduced class of SCC-recursive semantics [5], namely a parametric family of semantics which has been shown to represent a quite general well-founded scheme where specific proposals, including all traditional semantics mentioned above, can be placed. In this context we obtain a characterization of some cases of agreement, by exploiting the decomposition of the defeat graph into strongly connected components.

The paper is organized as follows. After reviewing the necessary basic concepts in Section 2, the notions of strongly connected component (SCC) and SCC-recursiveness are introduced in Section 3. In section 4 the definition of $CF2$ semantics is recalled and a property of its extensions, as significant for the sequel of the paper, is proved. The issues of agreement with grounded and stable semantics are dealt with in Sections 5 and 6 respectively. Finally Section 7 concludes the paper.

2 Basic concepts

The present work lies in the frame of the general theory of abstract argumentation frameworks proposed by Dung [1].

Definition 1. *An argumentation framework is a pair* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$*, where* $\mathcal{A}$ *is a set, and* $\rightarrow \subseteq (\mathcal{A} \times \mathcal{A})$ *is a binary relation on* $\mathcal{A}$*, called attack relation.*

In the following we will always assume that $\mathcal{A}$ is finite. An argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ can be represented as a directed graph, called *defeat graph*, where nodes are the arguments and edges correspond to the elements of the attack relation. In the following, the nodes that attack a given argument α are called *defeaters* or *parents* of α and form a set which is denoted as $\mathrm{par}_{\mathrm{AF}}(\alpha)$.

Definition 2. *Given an argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ *and a node* $\alpha \in \mathcal{A}$*, we define* $\mathrm{par}_{\mathrm{AF}}(\alpha) = \{\beta \in \mathcal{A} \mid \beta \rightarrow \alpha\}$*. If* $\mathrm{par}_{\mathrm{AF}}(\alpha) = \emptyset$*, then* α *is called an* initial *node.*

Since we will frequently consider properties of sets of arguments, it is useful to extend to them the notations defined for the nodes.

Definition 3. *Given an argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$*, a node* $\alpha \in \mathcal{A}$ *and two sets* $S, P \subseteq \mathcal{A}$*, we define:*

$$S \rightarrow \alpha \;\equiv\; \exists \beta \in S : \beta \rightarrow \alpha$$
$$\alpha \rightarrow S \;\equiv\; \exists \beta \in S : \alpha \rightarrow \beta$$
$$S \rightarrow P \;\equiv\; \exists \alpha \in S, \beta \in P : \alpha \rightarrow \beta$$

Two particular kinds of elementary argumentation frameworks need to be introduced as they will play some role in the following. The *empty argumentation framework*, denoted as $\mathrm{AF}_\emptyset$, is simply defined as $\mathrm{AF}_\emptyset = \langle \emptyset, \emptyset \rangle$. Furthermore, an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ is *monadic* if $|\mathcal{A}| = 1$ and $\rightarrow = \emptyset$.

The notion of self-defeating argument will be used too.

Definition 4. *Given an argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ *an argument* $\alpha \in \mathcal{A}$ *is* self-defeating *if* $\alpha \rightarrow \alpha$*. An argumentation framework* AF *is* free of self-defeating arguments *if* $\nexists \alpha \in \mathcal{A}$ *such that* $\alpha \rightarrow \alpha$*.*

We will also consider the *restriction* of an argumentation framework to a given subset of its nodes:

Definition 5. *Let* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ *be an argumentation framework, and let* $S \subseteq \mathcal{A}$ *be a set of arguments. The* restriction *of* AF *to* S *is the argumentation framework* $\mathrm{AF}{\downarrow_S} = \langle S, \rightarrow \cap (S \times S) \rangle$*.*

In Dung's theory, an argumentation semantics is defined by specifying the criteria for deriving, given a generic argumentation framework, the set of all possible extensions, each one representing a set of arguments considered to be acceptable together. Accordingly, a basic requirement for any extension E is that it is *conflict-free*, namely $\nexists \alpha, \beta \in E : \alpha \rightarrow \beta$. All argumentation semantics proposed in the literature satisfy this fundamental *conflict-free property*.

Given a generic argumentation semantics $\mathcal{S}$, the set of extensions prescribed by $\mathcal{S}$ for a given argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ is denoted as $\mathcal{E}_\mathcal{S}(\mathrm{AF})$. If it holds that $\forall \mathrm{AF}\ |\mathcal{E}_\mathcal{S}(\mathrm{AF})| = 1$, then the semantics $\mathcal{S}$ is said to follow the *unique-status approach*, otherwise it is said to follow the *multiple-status approach* [6]. We will say that two semantics $\mathcal{S}_1$ and $\mathcal{S}_2$ are in agreement on an argumentation framework AF if $\mathcal{E}_{\mathcal{S}_1}(\mathrm{AF}) = \mathcal{E}_{\mathcal{S}_2}(\mathrm{AF})$.

3 Strongly connected components and SCC-recursiveness

SCC-recursiveness is a property of (the extensions prescribed by) a semantics based on the graph theoretical notion of *strongly connected components* (SCCs).

Definition 6. *Given an argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$*, the binary relation of path-equivalence between nodes, denoted as* $PE_{\mathrm{AF}} \subseteq (\mathcal{A} \times \mathcal{A})$*, is defined as follows:*

– $\forall \alpha \in \mathcal{A}, (\alpha, \alpha) \in PE_{\mathrm{AF}}$

– *given two distinct nodes* $\alpha, \beta \in \mathcal{A}$, $(\alpha, \beta) \in PE_{\mathrm{AF}}$ *if and only if there is a path from* α *to* β *and a path from* β *to* α.

The *strongly connected components* of AF are the equivalence classes of nodes under the relation of path-equivalence. The set of the SCCs of AF is denoted as $\mathrm{SCCS}_{\mathrm{AF}}$. In the case of the empty argumentation framework, we assume $\mathrm{SCCS}_{\mathrm{AF}_\emptyset} = \{\emptyset\}$. Moreover, a strongly connected component $S \in \mathrm{SCCS}_{\mathrm{AF}}$ will be said to be monadic if $\mathrm{AF}{\downarrow}_S$ is monadic.

We extend to SCCs the notion of parents, namely the set of the other SCCs that attack a SCC S, which is denoted as $\mathrm{sccpar}_{\mathrm{AF}}(S)$, and we introduce the definition of *proper ancestors*, denoted as $\mathrm{sccanc}_{\mathrm{AF}}(S)$:

Definition 7. *Given an argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ *and a SCC* $S \in \mathrm{SCCS}_{\mathrm{AF}}$, *we define*

$$\mathrm{sccpar}_{\mathrm{AF}}(S) = \{P \in \mathrm{SCCS}_{\mathrm{AF}} \mid P \neq S \text{ and } P \rightarrow S\}$$

and

$$\mathrm{sccanc}_{\mathrm{AF}}(S) = \mathrm{sccpar}_{\mathrm{AF}}(S) \cup \bigcup_{P \in \mathrm{sccpar}_{\mathrm{AF}}(S)} \mathrm{sccanc}_{\mathrm{AF}}(P)$$

A SCC S such that $\mathrm{sccpar}_{\mathrm{AF}}(S) = \emptyset$ is called *initial*. The set of initial SCCs of AF, as it is easy to see, is non-empty and is denoted as $\mathcal{IS}(\mathrm{AF})$. The set of nodes of initial strongly connected components of AF is denoted as $IN(\mathrm{AF}) = \bigcup_{S \in \mathcal{IS}(\mathrm{AF})} S$.

It is well-known [7] that the graph obtained by considering SCCs as single nodes is acyclic, in other words SCCs can be partially ordered according to the relation of attack. This fact lies at the heart of the definition of SCC-recursiveness, which is based on the intuition that extensions can be built incrementally starting from initial SCCs and following the above mentioned partial order. In other words, the choices concerning extension construction carried out in an initial SCC do not depend on the choices concerning any other SCC, while they directly affect the choices about the subsequent SCCs and so on. While the basic underlying intuition is rather simple, the formalization of SCC-recursiveness is admittedly rather complex and involves some additional notions. Due to space limitations, we can only give here a quick account, while referring the reader to [5] for more details and examples. First of all, the choices (represented in the following definition by the set E, corresponding to a specific extension) in the antecedent SCCs determine a partition of the nodes of a set S (typically representing one or more subsequent SCCs) into three subsets:

Definition 8. *Given an argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, *a set* $E \subseteq \mathcal{A}$ *and a set* $S \subseteq \mathcal{A}$, *we define:*

– $D_{\mathrm{AF}}(S, E) = \{\alpha \in S \mid (E \setminus S) \rightarrow \alpha\}$
– $P_{\mathrm{AF}}(S, E) = \{\alpha \in S \mid (E \setminus S) \not\rightarrow \alpha \ \wedge \ \exists \beta \notin S : \beta \rightarrow \alpha \wedge E \not\rightarrow \beta\}$
– $U_{\mathrm{AF}}(S, E) = S \setminus (D_{\mathrm{AF}}(S, E) \cup P_{\mathrm{AF}}(S, E)) =$
$= \{\alpha \in S \mid (E \setminus S) \not\rightarrow \alpha \wedge \forall \beta \notin S : \beta \rightarrow \alpha \ E \rightarrow \beta\}$

Definition 8 is a slightly generalized version of the corresponding Definition 18 of [5]. In words, the set $D_{\mathrm{AF}}(S, E)$ consists of the nodes of S attacked by E from outside S, the set $U_{\mathrm{AF}}(S, E)$ includes any node α of S that is not attacked by E from outside S and is defended by E (i.e. the defeaters of α from outside S are all attacked by E), and $P_{\mathrm{AF}}(S, E)$ includes any node α of S that is not attacked by E from outside S and is not defended by E (i.e. at least one of the defeaters of α from outside S is not attacked by E). It is easy to verify that, when S is a SCC, as in the original Definition 18 of [5], $D_{\mathrm{AF}}(S, E)$, $P_{\mathrm{AF}}(S, E)$ and $U_{\mathrm{AF}}(S, E)$ are determined only by the elements of E that belong to the SCCs in $\mathrm{sccanc}_{\mathrm{AF}}(S)$.

Regarding E as a part of an extension which is being constructed, the idea is then that arguments in $D_{\mathrm{AF}}(S, E)$, being attacked by nodes in E, cannot be chosen in the construction of the extension E (i.e. do not belong to $E \cap S$). Selection of arguments to be included in E is therefore restricted to $(S \setminus D_{\mathrm{AF}}(S, E)) = (U_{\mathrm{AF}}(S, E) \cup P_{\mathrm{AF}}(S, E))$, which, for ease of notation, will be denoted in the following as $UP_{\mathrm{AF}}(S, E)$. On this basis and taking also into account the reinstatement principle [6, 2], we require the selection of nodes within a SCC S to be carried out on the restricted argumentation framework $\mathrm{AF}{\downarrow}_{UP_{\mathrm{AF}}(S,E)}$ without taking into account the attacks coming from $D_{\mathrm{AF}}(S, E)$.

Combining these ideas and skipping some details not strictly necessary in the context of the present paper, we can finally recall the definition of *SCC-recursiveness*:

Definition 9. *A given argumentation semantics* $\mathcal{S}$ *is* SCC-recursive *if and only if for any argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, $\mathcal{E}_{\mathcal{S}}(\mathrm{AF}) = \mathcal{GF}(\mathrm{AF}, \mathcal{A})$, *where for any* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ *and for any set* $C \subseteq \mathcal{A}$, *the function* $\mathcal{GF}(\mathrm{AF}, C) \subseteq 2^{\mathcal{A}}$ *is defined as follows:*
for any $E \subseteq \mathcal{A}$, $E \in \mathcal{GF}(\mathrm{AF}, C)$ *if and only if*

- *in case* $|\mathrm{SCCS}_{\mathrm{AF}}| = 1$, $E \in \mathcal{BF}_{\mathcal{S}}(\mathrm{AF}, C)$
- *otherwise,* $\forall S \in \mathrm{SCCS}_{\mathrm{AF}}$
  $(E \cap S) \in \mathcal{GF}(\mathrm{AF}{\downarrow}_{UP_{\mathrm{AF}}(S,E)}, U_{\mathrm{AF}}(S, E) \cap C)$

where $\mathcal{BF}_{\mathcal{S}}(\mathrm{AF}, C)$ *is a function, called* base function*, that, given an argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ *such that* $|\mathrm{SCCS}_{\mathrm{AF}}| = 1$ *and a set* $C \subseteq \mathcal{A}$, *gives a subset of* $2^{\mathcal{A}}$.

The base function $\mathcal{BF}_{\mathcal{S}}$ of a SCC-recursive semantics $\mathcal{S}$ is said to be conflict-free if $\forall \mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ and $\forall C \subseteq \mathcal{A}$ each element of $\mathcal{BF}_{\mathcal{S}}(\mathrm{AF}, C)$ is conflict free. It is known from Theorem 48 of [5] that if $\mathcal{BF}_{\mathcal{S}}$ is conflict-free, then any $E \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF})$ is conflict free for any AF.

Since Definition 9 is somewhat arduous to examine in its full detail, we just give some "quick and dirty" indications which are useful for the sequel of the paper (in particular, we do not consider the meaning of the parameter C in the description, as not necessary for the comprehension of this paper). The set of extensions $\mathcal{E}_{\mathcal{S}}(\mathrm{AF})$ of an argumentation framework AF is given by $\mathcal{GF}(\mathrm{AF}, \mathcal{A})$, namely by the invocation of the function $\mathcal{GF}$ which receives as parameters an argumentation framework (in this case the whole AF) and a set of arguments (in this case the whole $\mathcal{A}$). The function $\mathcal{GF}(\mathrm{AF}, C)$ is defined recursively. The base of the recursion is reached when AF consists of a unique SCC: in this case the set of extensions is directly given by the invocation of a semantics-specific base function $\mathcal{BF}_{\mathcal{S}}(\mathrm{AF}, C)$. In the other case, for each SCC S of AF the function $\mathcal{GF}$ is invoked recursively on the restriction $\mathrm{AF}{\downarrow}_{UP_{\mathrm{AF}}(S,E)}$. Note that the restriction concerns $UP_{\mathrm{AF}}(S, E)$, namely the part of S which "survives" the attacks of the preceding SCCs in the partial order.

The definition has also a constructive interpretation, which suggests an effective (recursive) procedure for computing all the extensions of an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ once a specific base function characterizing the semantics is assigned. A particular role in this context is played by the initial SCCs. In fact, for any initial SCC I, since by definition there are no outer attacks, the set of defended nodes coincides with I, i.e. $UP_{\mathrm{AF}}(I, E) = U_{\mathrm{AF}}(I, E) = I$ for any E. This gives rise to the invocation $\mathcal{GF}(\mathrm{AF}{\downarrow}_I, I)$ for any initial SCC I. Since $\mathrm{AF}{\downarrow}_I$ obviously consists of a unique SCC, according to Definition 9 the base function $\mathcal{BF}_{\mathcal{S}}(\mathrm{AF}{\downarrow}_I, I)$ is invoked, which returns the extensions of $\mathrm{AF}{\downarrow}_I$ according to the semantics $\mathcal{S}$. Therefore, the base function can be first computed on the initial SCCs, where it directly returns the extensions prescribed by the semantics. Then, the results of this computation are used to identify, within the subsequent SCCs, the restricted argumentation frameworks on which the procedure is recursively invoked.

All SCC-recursive semantics "share" this general scheme and only differ by the specific base function adopted. It has been shown [5] that all traditional semantics encompassed by Dung's framework (namely grounded, stable, complete, and preferred semantics) are SCC-recursive and the relevant base functions have been identified. In the following we will assume a basic knowledge of grounded semantics, denoted as $\mathcal{GR}$, stable semantics, denoted as $\mathcal{ST}$, and preferred semantics, denoted as $\mathcal{PR}$. We need to recall here only the formulation of the base function of grounded semantics (Proposition 44 of [5]):

Proposition 1. *For any argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ *such that* $|\mathrm{SCCS}_{\mathrm{AF}}| = 1$*, and for any* $C \subseteq \mathcal{A}$*, we have that*

$$\mathcal{BF}_{\mathcal{GR}}(\mathrm{AF}, C) = \begin{cases} \{\{\alpha\}\}, & \text{if } C = \mathcal{A} = \{\alpha\} \text{ and } \rightarrow = \emptyset; \\ \{\emptyset\}, & \text{otherwise.} \end{cases}$$

It is well-known that grounded semantics belongs to the unique-status approach. In the following we will denote the grounded extension of an argumentation framework AF as GE(AF).

4 A property of $\boldsymbol{CF2}$ semantics

Besides encompassing many significant previous proposals, the SCC-recursive scheme allows the definition of novel semantics in a relatively easy way. Examples of non-traditional SCC-recursive semantics and their properties are discussed in

[5], the most significant among them being $CF2$ semantics. In fact, $CF2$ semantics exhibits rather interesting properties (in particular a "symmetric" treatment of odd- and even-length cycles [8]) while its base function is particularly simple: $\mathcal{BF}_{CF2}(\mathrm{AF}, C) = \mathcal{MCF}_{\mathrm{AF}}$, where $\mathcal{MCF}_{\mathrm{AF}}$ denotes the set made up of all the maximal conflict-free sets of AF (note that the parameter C of Definition 9 plays no role at all in this case).

As a first contribution of this paper, here we provide the proof of an important property of $CF2$ semantics which, in particular, will be useful for the characterization of the cases of agreement between $CF2$ and grounded semantics. In words, we will show that any extension prescribed by $CF2$ semantics for an argumentation framework AF is a maximal conflict free set of AF.

A preliminary Lemma is needed.

Lemma 1. *Given an argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ *and a conflict free set* $E \subseteq \mathcal{A}$, $E \in \mathcal{MCF}_{\mathrm{AF}} \Leftrightarrow \forall \alpha \in \mathcal{A}$ *such that* $\alpha \not\rightarrow \alpha$, *the following disjunction of mutually exclusive conditions holds:* $\alpha \in E \vee E \rightarrow \alpha \vee \alpha \rightarrow E$.

Proof. $\Rightarrow$. Assume that $\exists \alpha \in \mathcal{A}$, $\alpha \not\rightarrow \alpha$, such that none of the three condition stated above holds. Then $\alpha \notin E \wedge E \not\rightarrow \alpha \wedge \alpha \not\rightarrow E$, which implies that $E \cup \{\alpha\}$ is conflict-free and a strict superset of E. But this contradicts the hypothesis that $E \in \mathcal{MCF}_{\mathrm{AF}}$.
$\Leftarrow$. Conversely assume that $E \notin \mathcal{MCF}_{\mathrm{AF}}$, then $\exists \alpha \in \mathcal{A}$ such that $\alpha \notin E$ and $E \cup \{\alpha\}$ is conflict-free, namely $\alpha \not\rightarrow \alpha$, $\alpha \notin E \wedge E \not\rightarrow \alpha \wedge \alpha \not\rightarrow E$, contradicting the hypothesis that one of the three conditions above holds.

Proposition 2. *For any argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, $\mathcal{E}_{CF2}(\mathrm{AF}) \subseteq \mathcal{MCF}_{\mathrm{AF}}$.

Proof. Since the base function of $CF2$ semantics is conflict-free, we know that any $E \in \mathcal{E}_{CF2}(\mathrm{AF})$ is conflict free. We have now to prove that it is maximal. First, recall that instantiating Definition 9 in the case of $CF2$ semantics we obtain: $E \in \mathcal{E}_{CF2}(\mathrm{AF})$ if and only if

- in case $|\mathrm{SCCS}_{\mathrm{AF}}| = 1$, $E \in \mathcal{MCF}_{\mathrm{AF}}$
- otherwise, $\forall S \in \mathrm{SCCS}_{\mathrm{AF}} (E \cap S) \in \mathcal{E}_{CF2}(\mathrm{AF}{\downarrow}_{UP_{\mathrm{AF}}(S,E)})$

If $|\mathrm{SCCS}_{\mathrm{AF}}| = 1$, $\mathcal{E}_{CF2}(\mathrm{AF}) = \mathcal{MCF}_{\mathrm{AF}}$ by definition and the thesis trivially follows.

Consider now the case $|\mathrm{SCCS}_{\mathrm{AF}}| > 1$ and assume recursively that $\forall S \in \mathrm{SCCS}_{\mathrm{AF}}$ $\forall E \in \mathcal{E}_{CF2}(\mathrm{AF})$ $(E \cap S) \in \mathcal{MCF}_{\mathrm{AF}{\downarrow}_{UP_{\mathrm{AF}}(S,E)}}$: we need to prove that $E \in \mathcal{MCF}_{\mathrm{AF}}$. Suppose by contradiction that $E \notin \mathcal{MCF}_{\mathrm{AF}}$. By Lemma 1 the following condition (i) holds: $\exists \alpha \in \mathcal{A} : \alpha \not\rightarrow \alpha$, $\alpha \notin E \wedge E \not\rightarrow \alpha \wedge \alpha \not\rightarrow E$. Now consider the strongly connected component $S \in \mathrm{SCCS}_{\mathrm{AF}}$ such that $\alpha \in S$. Since $E \not\rightarrow \alpha$ it is the case that $\alpha \in UP_{\mathrm{AF}}(S, E)$. By the inductive hypothesis $(E \cap S) \in \mathcal{MCF}_{\mathrm{AF}{\downarrow}_{UP_{\mathrm{AF}}(S,E)}}$, which, by Lemma 1 applied to $\mathrm{AF}{\downarrow}_{UP_{\mathrm{AF}}(S,E)}$, entails that the following disjunction holds: $\alpha \in (E \cap S) \vee (E \cap S) \rightarrow \alpha \vee \alpha \rightarrow (E \cap S)$. This clearly implies the following condition in AF: $\alpha \in E \vee E \rightarrow \alpha \vee \alpha \rightarrow E$. However, this is absurd since it contradicts condition (i) above.

5 Agreement with grounded semantics

Grounded semantics [9, 1] plays an important role in argumentation theory as it features desirable properties, such as conceptual clarity and computational tractability. Moreover, it is often regarded as a paradigmatic unique-status sceptical approach that can be used as a reference to evaluate other semantics. For these reasons the issue of agreement with grounded semantics is particularly significant and has been first considered in [1], where it is shown that a sufficient condition for agreement between grounded, preferred and stable semantics is that the argumentation framework is well-founded.

Definition 10. *(Definition 29 of [1]) An argumentation framework is well-founded iff there exists no infinite sequence* $\alpha_0, \alpha_1, \ldots, \alpha_n, \ldots$ *of (not necessarily distinct) arguments such that for each* i*,* α_{i+1} *attacks* α_i*.*

In the case of a finite argumentation framework, well-foundedness coincides with acyclicity of the defeat graph. We now consider the problem of agreement with grounded semantics in the generalized context of SCC-recursive semantics.

5.1 Determined argumentation frameworks

We will show that a complete agreement among SCC-recursive semantics holds if and only if the considered argumentation framework is *determined*.

Definition 11. *An argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ *is* determined *if and only if* $\nexists \alpha \in \mathcal{A} : \alpha \notin \mathrm{GE(AF)} \wedge \mathrm{GE(AF)} \not\rightarrow \alpha$.

In words, an argumentation framework AF is determined if and only if there are no "provisionally defeated" arguments in AF according to grounded semantics, i.e. the grounded extension is also a stable extension. Note that the empty argumentation framework is determined.

The set of determined argumentation frameworks, denoted as $\mathcal{DET}$, is of special interest because for any SCC-recursive semantics $\mathcal{S}$ respecting an obvious condition on the treatment of monadic argumentation frameworks it holds that $\mathcal{E}_{\mathcal{S}}(\mathrm{AF}) = \{\mathrm{GE(AF)}\}$ for any argumentation framework $\mathrm{AF} \in \mathcal{DET}$. In other words, a very comprehensive family of "reasonable" semantics show a uniform single-status behavior on these argumentation frameworks.

Proposition 3. *Let* $\mathcal{S}$ *be a SCC-recursive semantics identified by a conflict-free base function* $\mathcal{BF}_{\mathcal{S}}$ *such that*

$$\mathcal{BF}_{\mathcal{S}}(\langle \{\alpha\}, \emptyset \rangle, \{\alpha\}) = \{\{\alpha\}\}$$

(such a SCC-recursive semantics will be called grounded-compatible*).*

For any argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle \in \mathcal{DET}$ *it holds that* $\mathcal{E}_{\mathcal{S}}(\mathrm{AF}) = \{\mathrm{GE(AF)}\}$.

Proof. The proof immediately follows from the fact that for any such SCC-recursive semantics it holds that $\forall E \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF})$, $\mathrm{GE}(\mathrm{AF}) \subseteq E$ (Proposition 51 of [5]) and E is conflict-free. Since $\mathrm{AF} \in \mathcal{DET}$ $\forall \alpha \notin \mathrm{GE}(\mathrm{AF})$ it necessarily holds that $\mathrm{GE}(\mathrm{AF}) \rightarrow \alpha$, and therefore $\alpha \notin E$. As a consequence, only the case $E = \mathrm{GE}(\mathrm{AF})$ is possible.

It is also immediate to note that no argumentation framework outside $\mathcal{DET}$ features this property, namely, for any $\mathrm{AF} \notin \mathcal{DET}$ there is a grounded-compatible SCC-recursive semantics $\mathcal{S}$ such that $\mathcal{E}_{\mathcal{S}}(\mathrm{AF}) \neq \{\mathrm{GE}(\mathrm{AF})\}$, namely stable semantics.

Well-founded argumentation frameworks [1] are a special case of determined argumentation frameworks. In fact, if no cycles are present, all SCCs in AF consist of a single node and it is then easy to see that $\mathrm{AF} \in \mathcal{DET}$. On the other hand, the absence of cycles is a sufficient but not necessary topological condition for $\mathrm{AF} \in \mathcal{DET}$. Actually the absence of cycles is necessary only in the initial SCCs (which need to be monadic), and then recursively in the initial SCCs of the restricted argumentation framework obtained by taking into account that the nodes corresponding to the initial SCCs are necessarily included in any extension. This observation gives rise to a characterization of determined argumentation frameworks.

Definition 12. *An argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ *is initial-acyclic if* $\mathrm{AF} = \mathrm{AF}_{\emptyset}$ *or the following condition holds:* $\forall S \in \mathcal{IS}(\mathrm{AF})$ S *is monadic and* $\mathrm{AF}{\downarrow}_{UP_{\mathrm{AF}}((\mathcal{A} \setminus IN(\mathrm{AF})), IN(\mathrm{AF}))}$ *is initial-acyclic.*

The base of this recursive definition is represented by the empty argumentation framework. The recursion is well-founded as the set $IN(\mathrm{AF})$ is non-empty for a non-empty argumentation framework, which means that at each recursive step an argumentation framework with a strictly lesser number of nodes is considered. The set of initial-acyclic argumentation frameworks is denoted by $\mathcal{IAA}$. The following proposition shows that $\mathcal{IAA} = \mathcal{DET}$.

Proposition 4. *For any argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, $\mathrm{AF} \in \mathcal{IAA}$ *if and only if* $\mathrm{AF} \in \mathcal{DET}$.

Proof. Let us first show that if $\mathrm{AF} \in \mathcal{IAA}$ then the grounded extension is also stable. It is known [1] that, for any finite AF, $\mathrm{GE}(\mathrm{AF}) = \bigcup_{i \geq 1} \mathrm{F}^{i}_{\mathrm{AF}}(\emptyset)$, where, given a set $S \subseteq \mathcal{A}$, $\mathrm{F}_{\mathrm{AF}}(S) = \{\alpha \in \mathcal{A} : \forall \beta \in \mathrm{par}_{\mathrm{AF}}(\alpha), S \rightarrow \beta\}$, $\mathrm{F}^{1}_{\mathrm{AF}}(S) = \mathrm{F}_{\mathrm{AF}}(S)$, and $\mathrm{F}^{i}_{\mathrm{AF}}(S) = \mathrm{F}_{\mathrm{AF}}(\mathrm{F}^{i-1}_{\mathrm{AF}}(S))$. Now, since $\mathrm{AF} \in \mathcal{IAA}$, it holds that $\mathrm{F}^{1}_{\mathrm{AF}}(\emptyset) = IN(\mathrm{AF})$. After suppressing the arguments attacked by arguments in $IN(\mathrm{AF})$ we obtain $\mathrm{AF}' = \mathrm{AF}{\downarrow}_{UP_{\mathrm{AF}}((\mathcal{A} \setminus IN(\mathrm{AF})), IN(\mathrm{AF}))}$. Now, if AF' is empty the statement is proved, since any argument of AF is either included in or attacked by GE(AF). Otherwise we have, by hypothesis, that all initial strongly connected components of AF' are monadic. This entails that all their nodes belong to $\mathrm{F}^{2}_{\mathrm{AF}}(\emptyset)$ and therefore to GE(AF). Iterating the same reasoning as above we obtain a restricted argumentation framework AF'', and so on until we reach the case of an empty restricted argumentation framework. Since any node considered at any step is either included in or attacked by GE(AF), it turns out that AF $\in \mathcal{DET}$.

Turning to the other part of the proof, let us show that if AF $\notin \mathcal{IAA}$ then AF $\notin \mathcal{DET}$. Let us first consider the case where some initial strongly connected component of AF is not monadic, then its elements are not included in nor attacked by GE(AF) and therefore AF $\notin \mathcal{DET}$. Otherwise with a similar reasoning as in the first part of the proof, we are lead to consider a sequence of restricted argumentation frameworks. Since at least one of them does not belong to $\mathcal{IAA}$, it turns out as before that some of its nodes are not included in nor attacked by GE(AF) and the conclusion follows.

5.2 Almost determined argumentation frameworks

While only determined argumentation frameworks ensure complete agreement among all grounded-compatible SCC-recursive semantics, it can be observed that there is a larger class of argumentation frameworks where an almost complete agreement is reached. Consider for instance the case of an argumentation framework consisting just of a self-defeating argument, namely AF $= \langle \{\alpha\}, \{(\alpha, \alpha)\} \rangle$. In this case we have that $\mathcal{E}_{\mathcal{GR}}(\mathrm{AF}) = \{\emptyset\}$ and, in virtue of the conflict-free property, for any semantics $\mathcal{S}$ which admits extensions on AF it must also hold that $\mathcal{E}_{\mathcal{S}}(\mathrm{AF}) = \{\emptyset\}$. However, since stable semantics is unable to prescribe estensions in this case, $\mathcal{E}_{\mathcal{ST}}(\mathrm{AF}) = \emptyset \neq \{\emptyset\}$. In this case, disagreement arises from the non-existence of stable extensions rather than from the existence of extensions different from GE(AF). Therefore, excluding AF from the set of argumentation frameworks where semantics agree might be considered a little bit questionable and/or misleading, since, actually, all semantics able to prescribe extensions for AF are in agreement.

On the basis of this observation, it is useful to consider the question of agreement focusing on those semantics that are *universally defined*.

Definition 13. *An argumentation semantics $\mathcal{S}$ is universally defined if for any argumentation framework* AF $\mathcal{E}_{\mathcal{S}}(\mathrm{AF}) \neq \emptyset$.

As to our knowledge, stable semantics is the only example in the literature of a semantics which is not universally defined.

As shown by the simple example above, the set of argumentation frameworks where universally defined semantics agree is larger than $\mathcal{DET}$: we will now characterize this class of argumentation frameworks, called *almost determined*.

Definition 14. *An argumentation framework* AF $= \langle \mathcal{A}, \rightarrow \rangle$ *is* almost determined *if and only if for any* $\alpha \in \mathcal{A}$, $(\alpha \notin \mathrm{GE(AF)} \wedge \mathrm{GE(AF)} \not\rightarrow \alpha) \Rightarrow (\alpha, \alpha) \in \rightarrow$.

In words, an argumentation framework is almost determined if all the nodes which are not attacked nor included in the grounded extension are self-defeating. The set of almost determined argumentation frameworks will be denoted as $\mathcal{AD}$. Clearly $\mathcal{DET} \subsetneq \mathcal{AD}$.

Proposition 5. *Let $\mathcal{S}$ be a universally defined and grounded compatible SCC-recursive semantics identified by a conflict-free base function $\mathcal{BF}_{\mathcal{S}}$. For any argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle \in \mathcal{AD}$ *it holds that* $\mathcal{E}_{\mathcal{S}}(\mathrm{AF}) = \{\mathrm{GE}(\mathrm{AF})\}$.

Proof. We know that, since $\mathcal{BF}_{\mathcal{S}}$ is conflict-free, for any argumentation framework AF, $\forall E \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF})$ E is conflict free. Then, the statement follows from the fact that $\forall E \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF})$, $\mathrm{GE}(\mathrm{AF}) \subseteq E$, which entails that the arguments attacked by the grounded extension are also attacked by any other extension. Therefore only arguments not included in and not attacked by GE(AF) can belong to $E \setminus \mathrm{GE}(\mathrm{AF})$. However, by hypothesis such arguments are self-defeating and, since any extension E is conflict-free, can not belong to E.

The proposition above shows that agreement is ensured on almost determined argumentation frameworks for any SCC-recursive semantics which satisfies the three very reasonable properties of being universally defined, grounded compatible and conflict-free. We now also show that such an agreement can not be achieved outside the class of almost determined argumentation frameworks.

Proposition 6. *For any argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle \notin \mathcal{AD}$ *there is a universally defined and grounded compatible SCC-recursive semantics $\mathcal{S}$ identified by a conflict-free base function $\mathcal{BF}_{\mathcal{S}}$ such that* $\mathcal{E}_{\mathcal{S}}(\mathrm{AF}) \neq \{\mathrm{GE}(\mathrm{AF})\}$.

Proof. We prove that if $\mathrm{AF} \notin \mathcal{AD}$ then $\mathcal{E}_{CF2}(\mathrm{AF}) \neq \{\mathrm{GE}(\mathrm{AF})\}$. It is immediate to see that $CF2$ semantics is universally defined and grounded compatible and that its base function is conflict-free. By Proposition 2, $\mathcal{E}_{CF2}(\mathrm{AF}) \subseteq \mathcal{MCF}_{\mathrm{AF}}$, namely the extensions prescribed by $CF2$ semantics for an argumentation framework AF are maximal conflict free sets of AF. Now if $\mathrm{AF} \notin \mathcal{AD}$, $\exists \alpha \in \mathcal{A}$ such that α is not self-defeating, $\alpha \notin \mathrm{GE}(\mathrm{AF})$ and $\mathrm{GE}(\mathrm{AF}) \not\rightarrow \alpha$. This also implies $\alpha \not\rightarrow \mathrm{GE}(\mathrm{AF})$ due to the well-known property of admissibility of GE(AF) [1], namely $\alpha \rightarrow \mathrm{GE}(\mathrm{AF}) \Rightarrow \mathrm{GE}(\mathrm{AF}) \rightarrow \alpha$. Then, by Lemma 1, $\mathrm{GE}(\mathrm{AF}) \notin \mathcal{MCF}_{\mathrm{AF}}$ and necessarily $\mathcal{E}_{CF2}(\mathrm{AF}) \neq \{\mathrm{GE}(\mathrm{AF})\}$.

6 Agreement with stable semantics

Stable semantics represents a traditional and intuitively simple proposal among multiple-status approaches: a stable extension is simply a conflict-free set which attacks all arguments not included in it. For this reason, agreement with stable semantics represents a sort of uncontroversial situation where no argument is left in a sort of "undecided" status. In [1] an argumentation framework AF such that preferred and stable semantics are in agreement is said to be *coherent*. Here we will characterize a family of argumentation frameworks, called SCC-symmetric, where agreement is ensured for a class of multiple-status semantics including stable, preferred and $CF2$ semantics.

First we need to introduce the notion of symmetric argumentation framework (slightly different from the one proposed in [4]), noting also that symmetry is preserved by the restriction operator.

Definition 15. *An argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ *is* symmetric *if for any* $\alpha, \beta \in \mathcal{A}$*,* $\alpha \rightarrow \beta \Leftrightarrow \beta \rightarrow \alpha$*.*

Lemma 2. *Given a symmetric argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ *and a set* $S \subseteq \mathcal{A}$*,* $\mathrm{AF}{\downarrow_S}$ *is symmetric.*

Proof. Let us consider two arguments α, β in $\mathrm{AF}{\downarrow_S}$ such that $\alpha \rightarrow \beta$. It is immediate to see that this relation also holds in AF and, since the latter is symmetric, $\beta \rightarrow \alpha$ in AF. Since $\alpha, \beta \in S$, $\beta \rightarrow \alpha$ also holds in $\mathrm{AF}{\downarrow_S}$.

As it will be more evident from Proposition 7, it is quite natural that extensions of a symmetric argumentation framework free of self-defeating arguments coincide with its maximal conflict free sets, if the multiple-status approach is adopted. Argumentation semantics satisfying this requirement will be called *-symmetric.

Definition 16. *An argumentation semantics* $\mathcal{S}$ *is *-symmetric if for any argumentation framework* AF *which is symmetric and free of self-defeating arguments* $\mathcal{E}_{\mathcal{S}}(\mathrm{AF}) = \mathcal{MCF}_{\mathrm{AF}}$*.*

As one may imagine, a SCC-recursive semantics is *-symmetric if and only if its base function has a *-symmetric behavior on single-SCC argumentation frameworks.

Lemma 3. *A SCC-recursive semantics* $\mathcal{S}$ *is *-symmetric if and only if, for any argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ *which is symmetric, free of self-defeating arguments and such that* $|\mathrm{SCCS_{AF}}| = 1$*,* $\mathcal{BF}_{\mathcal{S}}(\mathrm{AF}, \mathcal{A}) = \mathcal{MCF}_{\mathrm{AF}}$*.*

Proof. $\Rightarrow$. Assume that the base function satisfies the hypothesis and consider a generic argumentation framework AF which is symmetric and free of self-defeating arguments. Notice first that $\forall S \in \mathrm{SCCS_{AF}}\ \mathrm{sccpar_{AF}}(S) = \emptyset$, i.e. all of the strongly connected components are initial. In fact, given $S_1, S_2 \in \mathrm{SCCS_{AF}}$ such that $S_1 \rightarrow S_2$, since AF is symmetric also $S_2 \rightarrow S_1$ holds, entailing that all of the nodes of $S_1 \cup S_2$ are mutually reachable, i.e. $S_1 = S_2$. Then, $\forall S \in \mathrm{SCCS_{AF}}\ U_{\mathrm{AF}}(S, E) = UP_{\mathrm{AF}}(S, E) = S$, and it is easy to see that, according to Definition 9, $E \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF})$ if and only if $\forall S \in \mathrm{SCCS_{AF}}\ (E \cap S) = \mathcal{BF}_{\mathcal{S}}(\mathrm{AF}{\downarrow_S}, S)$. Now, $\forall S \in \mathrm{SCCS_{AF}}\ \mathrm{AF}{\downarrow_S}$ is free of self-defeating arguments and by Lemma 2 is also symmetric, thus by the hypothesis $\mathcal{BF}_{\mathcal{S}}(\mathrm{AF}{\downarrow_S}, S) = \mathcal{MCF}_{\mathrm{AF}{\downarrow_S}}$. In sum, we have that $E \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF})$ if and only if $\forall S \in \mathrm{SCCS_{AF}}\ (E \cap S) \in \mathcal{MCF}_{\mathrm{AF}{\downarrow_S}}$, and since all of the strongly connected components are initial the latter condition is equivalent to $E \in \mathcal{MCF}_{\mathrm{AF}}$.
$\Leftarrow$. Assuming by contradiction that the conclusion is not verified we are led to consider an argumentation framework AF, which is symmetric and free of self-defeating arguments, where $\mathcal{E}_{\mathcal{S}}(\mathrm{AF}) = \mathcal{BF}_{\mathcal{S}}(\mathrm{AF}, \mathcal{A}) \neq \mathcal{MCF}_{\mathrm{AF}}$, entailing that $\mathcal{S}$ is not *-symmetric.

Several significant multiple-status semantics, though their definition is based on quite different principles, share the property of being *-symmetric (a similar result is proved in [4]).

Proposition 7. *Stable semantics, preferred semantics and* $CF2$ *semantics are *-symmetric.*

Proof. According to Lemma 3, for any such semantics $\mathcal{S}$ we have to prove that, given an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ which is symmetric, free of self-defeating arguments and such that $|\mathrm{SCCS}_{\mathrm{AF}}| = 1$, $\mathcal{BF}_{\mathcal{S}}(\mathrm{AF}, \mathcal{A}) = \mathcal{MCF}_{\mathrm{AF}}$. For $CF2$ semantics this holds by definition. As for stable and preferred semantics, notice that, as $|\mathrm{SCCS}_{\mathrm{AF}}| = 1$, $\mathcal{BF}_{\mathcal{S}}(\mathrm{AF}, \mathcal{A}) = \mathcal{E}_{\mathcal{S}}(\mathrm{AF})$. Taking into account from [1] that $\mathcal{E}_{\mathcal{ST}}(\mathrm{AF}) \subseteq \mathcal{E}_{\mathcal{PR}}(\mathrm{AF})$, it is sufficient to prove that $\mathcal{MCF}_{\mathrm{AF}} \subseteq \mathcal{E}_{\mathcal{ST}}(\mathrm{AF})$ and that $\mathcal{E}_{\mathcal{PR}}(\mathrm{AF}) \subseteq \mathcal{MCF}_{\mathrm{AF}}$. First, let us consider a set $E \in \mathcal{MCF}_{\mathrm{AF}}$ and let us prove that it is a stable extension, i.e. that $\forall \alpha \notin E \; E \rightarrow \alpha$. Assuming by contradiction that $E \not\rightarrow \alpha$, since AF is symmetric also $\alpha \not\rightarrow E$ holds. Since α cannot be self-defeating by the hypothesis on AF, the set $E \cup \{\alpha\}$ is conflict-free, contradicting the fact that $E \in \mathcal{MCF}_{\mathrm{AF}}$. Let us turn now to the other inclusion condition, considering a set $E \in \mathcal{E}_{\mathcal{PR}}(\mathrm{AF})$ and assuming by contradiction that $E \notin \mathcal{MCF}_{\mathrm{AF}}$: since E is conflict-free, this entails that $\exists E' \subseteq \mathcal{MCF}_{\mathrm{AF}}$ such that $E \subsetneq E'$. However, by the first inclusion condition $E' \in \mathcal{E}_{\mathcal{PR}}(\mathrm{AF})$, contradicting the fact that E is a preferred extension.

In symmetric argumentation frameworks non-mutual attacks cannot exist: this seriously limits their applicability for modeling practical situations. Their properties however provide the basis for analyzing a more interesting family of argumentation frameworks called *SCC-symmetric*.

Definition 17. *An argumentation framework* AF *is SCC-symmetric if* $\forall S \in \mathrm{SCCS}_{\mathrm{AF}} \; \mathrm{AF}{\downarrow_S}$ *is symmetric.*

Definition 17 is equivalent to forbidding non-mutual attacks only within cycles.

Proposition 8. *An argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ *is SCC-symmetric if and only if for every cycle* $\alpha_0 \rightarrow \alpha_1 \rightarrow \ldots \rightarrow \alpha_n \rightarrow \alpha_0$ *it holds that* $\forall i \in \{1 \ldots n\} \; \alpha_i \rightarrow \alpha_{i-1}$.

Proof. As for the if part of the proof, notice that any two nodes $\alpha, \beta \in S$, such that $\alpha \neq \beta$ and $S \in \mathrm{SCCS}_{\mathrm{AF}}$, are mutually reachable, therefore in particular they belong to a cycle. As a consequence, if $\alpha \rightarrow \beta$ then by the hypothesis also $\beta \rightarrow \alpha$ holds. As for the other part of the proof, if α_i and α_{i-1} belong to a cycle then they are in the same strongly connected component, thus if $\alpha_{i-1} \rightarrow \alpha_i$ then by the SCC-symmetry of AF also $\alpha_i \rightarrow \alpha_{i-1}$ holds.

To prove, in Theorem 1, the main result about agreement in SCC-symmetric argumentation frameworks, we need a preliminary lemma concerning the SCC-recursive schema.

Lemma 4. *Given an SCC-recursive semantics* $\mathcal{S}$, $E \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF})$ *if and only if* $\forall S \in \mathrm{SCCS}_{\mathrm{AF}} \; (E \cap S) \in \mathcal{GF}_{\mathcal{S}}(\mathrm{AF}{\downarrow_{UP_{\mathrm{AF}}(S,E)}}, U_{\mathrm{AF}}(S,E))$*, where* $\mathcal{GF}_{\mathcal{S}}(\mathrm{AF}, C)$ *is a function specific for the semantics* $\mathcal{S}$*. Moreover,* $\forall \mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ *it holds that* $\mathcal{GF}_{\mathcal{S}}(\mathrm{AF}, \mathcal{A}) = \mathcal{E}_{\mathcal{S}}(\mathrm{AF})$.

Proof. By Definition 9, if $|\mathrm{SCCS_{AF}}| = 1$ then $\mathcal{BF_S}(\mathrm{AF}, \mathcal{A}) = \mathcal{GF}(\mathrm{AF}, \mathcal{A})$, which is also equal to $\mathcal{GF}(\mathrm{AF}{\downarrow}_{UP_{\mathrm{AF}}(S,E)}, U_{\mathrm{AF}}(S,E))$ since in this case $UP_{\mathrm{AF}}(S,E) = U_{\mathrm{AF}}(S,E) = \mathcal{A}$. From Definition 9 we have that $E \in \mathcal{E_S}(\mathrm{AF})$ if and only if $E \in \mathcal{GF}(\mathrm{AF}, \mathcal{A})$ and in case $|\mathrm{SCCS_{AF}}| = 1$ we can substitute $\mathcal{BF_S}(\mathrm{AF}, \mathcal{A})$ with the expression above. This yields $E \in \mathcal{GF}(\mathrm{AF}, \mathcal{A})$ if and only if $\forall S \in \mathrm{SCCS_{AF}} (E \cap S) \in \mathcal{GF}(\mathrm{AF}{\downarrow}_{UP_{\mathrm{AF}}(S,E)}, U_{\mathrm{AF}}(S,E))$. Then the conclusion easily follows by taking into account that $\mathcal{GF}$ actually depends (through $\mathcal{BF_S}$) on the specific semantics $\mathcal{S}$.

Theorem 1. *In any argumentation framework which is SCC-symmetric and free of self-defeating arguments all of the *-symmetric semantics are in agreement, i.e. they prescribe the same set of extensions.*

Proof. It is sufficient to show that, given an argumentation framework AF satisfying the hypothesis and two *-simmetric semantics $\mathcal{S}_1$ and $\mathcal{S}_2$, $\forall E \in \mathcal{E}_{\mathcal{S}_1}(\mathrm{AF})\ E \in \mathcal{E}_{\mathcal{S}_2}(\mathrm{AF})$ (the reverse condition can then be obtained by the same reasoning). According to Lemma 4, given $E \in \mathcal{E}_{\mathcal{S}_1}(\mathrm{AF})$, we have to prove that $\forall S \in \mathrm{SCCS_{AF}} (E \cap S) \in \mathcal{GF}_{\mathcal{S}_2}(\mathrm{AF}{\downarrow}_{UP_{\mathrm{AF}}(S,E)}, U_{\mathrm{AF}}(S,E))$. We reason by induction along the strongly connected components of the argumentation framework. In particular, at any step we consider a specific $S \in \mathrm{SCCS_{AF}}$ and we prove the following conditions:

1. $UP_{\mathrm{AF}}(S,E) = U_{\mathrm{AF}}(S,E)$ (i.e., $P_{\mathrm{AF}}(S,E) = \emptyset$)
2. $(E \cap S) \in \mathcal{GF}_{\mathcal{S}_2}(\mathrm{AF}{\downarrow}_{U_{\mathrm{AF}}(S,E)}, U_{\mathrm{AF}}(S,E)) = \mathcal{E}_{\mathcal{S}_2}(\mathrm{AF}{\downarrow}_{U_{\mathrm{AF}}(S,E)})$
3. $\mathcal{E}_{\mathcal{S}_2}(\mathrm{AF}{\downarrow}_{U_{\mathrm{AF}}(S,E)}) = \mathcal{E}_{\mathcal{ST}}(\mathrm{AF}{\downarrow}_{U_{\mathrm{AF}}(S,E)})$

assuming that these conditions hold for any $S' \in \mathrm{sccanc_{AF}}(S)$ (notice that the case $\mathrm{sccanc_{AF}}(S) = \emptyset$, i.e. S is an initial strongly connected component, is covered in the following proof). Then the conclusion is immediate from the first and second conditions.
As for the first condition (which is obvious when S is initial), we have to prove that $\forall \alpha \in UP_{\mathrm{AF}}(S,E)$ if $\beta \to \alpha$ and $\beta \notin S$ then $E \to \beta$. Notice that $\beta \in S'$ with $S' \in \mathrm{sccanc_{AF}}(S)$, and $\beta \notin E$ since $\alpha \notin D_{\mathrm{AF}}(S,E)$. Since by the first condition applied to S' $P_{\mathrm{AF}}(S',E) = \emptyset$, either $\beta \in D_{\mathrm{AF}}(S',E)$ or $\beta \in U_{\mathrm{AF}}(S',E)$. In the first case, $E \to \beta$ by definition. In the second case, since $(E \cap S') \in \mathcal{E}_{\mathcal{ST}}(\mathrm{AF}{\downarrow}_{U_{\mathrm{AF}}(S',E)})$ by the second and third conditions and $\beta \notin (E \cap S')$, it holds that $(E \cap S') \to \beta$, thus again $E \to \beta$.
Let us turn to the second condition. Since $E \in \mathcal{E}_{\mathcal{S}_1}(\mathrm{AF})$, according to Lemma 4 $(E \cap S) \in \mathcal{GF}_{\mathcal{S}_1}(\mathrm{AF}{\downarrow}_{UP_{\mathrm{AF}}(S,E)}, U_{\mathrm{AF}}(S,E))$, which by the above proof is equal to $\mathcal{GF}_{\mathcal{S}_1}(\mathrm{AF}{\downarrow}_{U_{\mathrm{AF}}(S,E)}, U_{\mathrm{AF}}(S,E))$, the latter being equal to $\mathcal{E}_{\mathcal{S}_1}(\mathrm{AF}{\downarrow}_{U_{\mathrm{AF}}(S,E)})$ by Lemma 4. Now, since AF is SCC-symmetric $\mathrm{AF}{\downarrow}_S$ is symmetric by definition, entailing by Lemma 2 that $\mathrm{AF}{\downarrow}_{U_{\mathrm{AF}}(S,E)}$ is symmetric in turn. Notice that this argumentation framework, as AF, is free of self-defeating arguments. Then, since both $\mathcal{S}_1$ and $\mathcal{S}_2$ are *-symmetric $\mathcal{E}_{\mathcal{S}_1}(\mathrm{AF}{\downarrow}_{U_{\mathrm{AF}}(S,E)}) = \mathcal{E}_{\mathcal{S}_2}(\mathrm{AF}{\downarrow}_{U_{\mathrm{AF}}(S,E)}) = \mathcal{MCF}_{\mathrm{AF}{\downarrow}_{U_{\mathrm{AF}}(S,E)}}$. In sum, $(E \cap S) \in \mathcal{E}_{\mathcal{S}_2}(\mathrm{AF}{\downarrow}_{U_{\mathrm{AF}}(S,E)})$, which by Lemma 4 is equal to $\mathcal{GF}_{\mathcal{S}_2}(\mathrm{AF}{\downarrow}_{U_{\mathrm{AF}}(S,E)}, U_{\mathrm{AF}}(S,E))$.

Finally, the third condition follows from Proposition 7, which states in particular that stable semantics is *-symmetric, entailing that $\mathcal{E}_{\mathcal{ST}}(\mathrm{AF}{\downarrow}_{U_{\mathrm{AF}}(S,E)}) = \mathcal{MCF}_{\mathrm{AF}{\downarrow}_{U_{\mathrm{AF}}(S,E)}} = \mathcal{E}_{\mathcal{S}_2}(\mathrm{AF}{\downarrow}_{U_{\mathrm{AF}}(S,E)})$.

The following result immediately follows from the previous theorem and Proposition 7.

Corollary 1. *For any argumentation framework* AF *which is SCC-symmetric and free of self-defeating arguments,* $\mathcal{E}_{\mathcal{PR}}(\mathrm{AF}) = \mathcal{E}_{CF2}(\mathrm{AF}) = \mathcal{E}_{\mathcal{ST}}(\mathrm{AF})$*, thus in particular* AF *is coherent.*

Theorem 1 and Corollary 1 generalize the results about agreement provided in [4], where only symmetric argumentation frameworks are considered (which, as already remarked, feature a limited expressivity since they prevent, for instance, that an initial argument attacks any other argument). Moreover, agreement is proved for a family of multiple-status SCC-recursive semantics, including the most significant literature proposals we are aware of.

In [1] it was shown that a sufficient condition for agreement between preferred and stable semantics is that the considered argumentation framework is *limited controversial*. A finite argumentation framework is limited controversial if it does not include any odd-length cycle. The classes of SCC-symmetric and limited controversial argumentation frameworks are non-disjoint but distinct. In fact, a SCC-symmetric argumentation framework may contain cycles of any length, while a limited controversial argumentation framework may consist, for instance, of an even-length cycle which is not symmetric.

It is interesting to note that the property of SCC-symmetry may be recovered from assumptions on the attack relation which have been previously considered in the literature and are not directly related to decomposition into SCCs. For instance in [10] the case is considered where conflicts among arguments arise only from contradicting conclusions, namely only the *rebutting* kind of defeat is allowed while *undercutting* defeat is not (we follow here the terminology of [9], note that the notion of rebutting defeat we adopt includes attack against subarguments, that some authors call instead undercut). It is shown in Proposition 26 of [10] that if only rebutting defeat is allowed, the defeat graph is SCC-symmetric (such a graph is called *r-type* in [10]). From another perspective, in [11] it is shown that when the attack relation results from a symmetric conflict relation and a transitive preference relation between arguments the defeat graph satisfies a property called *strict acyclicity*, which is actually equivalent to SCC-symmetry through the characterization given in Proposition 8.

7 Conclusions

In this paper we have analyzed the issue of characterizing argumentation frameworks where semantics agree, exploiting to this purpose the recently introduced notion of SCC-recursiveness and the relevant existing results. Focusing on the two traditional questions of agreement with grounded and stable semantics, some novel results have been obtained. As to the first question, the family of determined argumentation frameworks where any "reasonable" SCC-recursive semantics agrees with grounded semantics has been identified. Adding the requirement that the semantics is universally defined, a larger family of argumentation frameworks where such an agreement is ensured has been characterized. As to the second question, it has been shown that agreement is ensured, for a class of semantics including stable, preferred and $CF2$ semantics, on the significant family of SCC-symmetric argumentation frameworks. Among future work directions, we mention in particular the definition and study of forms of agreement at the level of justification states of arguments rather than of extensions.

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References

1. Dung, P.M.: On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming, and n-person games. Artificial Intelligence **77**(2) (1995) 321–357
2. Baroni, P., Giacomin, M.: Evaluation and comparison criteria for extension-based argumentation semantics. In Dunne, P.E., Bench-Capon, T., eds.: Proc. of the 1st International Conference on Computational Models of Arguments (COMMA 2006), Liverpool, UK, IOS Press (2006) 157–168
3. Caminada, M.: On the issue of reinstatement in argumentation. In: Proc. of the 10th European Conference on Logics in Artificial Intelligence (JELIA 06), Liverpool, UK, Springer (2006) 111–123
4. Coste-Marquis, S., Devred, C., Marquis, P.: Symmetric argumentation frameworks. In: Proceedings of the 8th European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty (ECSQARU 2005), Barcelona, E (2005) 317–328
5. Baroni, P., Giacomin, M., Guida, G.: Scc-recursiveness: a general schema for argumentation semantics. Artificial Intelligence **168**(1-2) (2005) 165–210
6. Prakken, H., Vreeswijk, G.A.W.: Logics for defeasible argumentation. In Gabbay, D.M., Guenthner, F., eds.: Handbook of Philosophical Logic, Second Edition. Kluwer Academic Publishers, Dordrecht (2001)
7. Cormen, T.H., Leiserson, C.E., Rivest, R.L., Stein, C.: Introduction to Algorithms. MIT Press and McGraw-Hill (2001)
8. Baroni, P., Giacomin, M.: Solving semantic problems with odd-length cycles in argumentation. In: Proceedings of the 7th European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty (ECSQARU 2003), Aalborg, Denmark, LNAI 2711, Springer-Verlag (2003) 440–451
9. Pollock, J.L.: How to reason defeasibly. Artificial Intelligence **57**(1) (1992) 1–42
10. Baroni, P., Giacomin, M., Guida, G.: Self-stabilizing defeat status computation: dealing with conflict management in multi-agent systems. Artificial Intelligence **165**(2) (2005) 187–259
11. Kaci, S., van der Torre, L.W.N., Weydert, E.: Acyclic argumentation: Attack = conflict + preference. In: Proceedings of the 17th European Conference on Artificial Intelligence (ECAI 2006), Riva del Garda, Italy, IOS Press (2006) 725–726